One method for solving three equations with three unknowns is stated below.

1	Number the equations in the system 1, 2, and 3.
2	Choose any variable to eliminate.
3	Pick any two of the equations from the original system and use the elimination method. Call the result equation #4.
4	Go back to the original system and pick **two different** equations from the system. Eliminate the **same** variable by using the elimination method. Call the result equation #5.
5	Use equations #4 and #5 together to eliminate a second variable by the elimination method. This will give you one of your unknowns.
6	Take your solution from step 5 and substitute back into equation #4 or #5 to find a second unknown.
7	Find your third unknown by plugging the two you have into an original equation.
8	Write your solution as an ordered triple (x,y,z)
9	Check your solution in the original system.

Solve the following systems of equations.

1. ① $x - y + z = -1$
   ② $x + y + 3z = -3$
   ③ $2x - y + 2z = 0$

eliminating y

① $x - y + z = -1$
② $+\ x + y + 3z = -3$
④ $2x + 4z = -4$

② $x + y + 3z = -3$
③ $+\ 2x - y + 2z = 0$
⑤ $3x + 5z = -3$

④ $\cdot 3$ $6x + 12z = -12$
⑤ $\cdot -2$ $+\ -6x - 10z = 6$
$2z = -6$
$z = -3$ plug into

$3x + 5(-3) = -3$
$3x - 15 = -3$
$3x = 12$
$x = 4$, $z = -3$ plug into

$2(4) - y + 2(-3) = 0$
$8 - y - 6 = 0$
$-y + 2 = 0$
$y = 2$

Solution: $(4, 2, -3)$

$$\begin{cases} x - 2y + 3z = 12 \\ 2x - y - 2z = 5 \\ 2x + 2y - z = 4 \end{cases}$$

eliminating y

① $x - 2y + 3z = 12$
③ $+\ 2x + 2y - z = 4$
④ $3x + 2z = 16$

2·② $4x - 2y - 4z = 10$
③ $+\ 2x + 2y - z = 4$
⑤ $6x - 5z = 14$

-2·④ $-6x - 4z = -32$
elim ⑤ $+\ 6x - 5z = 14$
$-9z = -18$
$z = 2$ → ④ $3x + 2(2) = 16$
$3x + 4 = 16$
$3x = 12$
$x = 4$

② $2(4) - y - 2(2) = 5$
$8 - y - 4 = 5$
$4 - y = 5$
$y = -1$

$(4, -1, 2)$

★ sub case

$$\begin{cases} x - 2y + z = -4 \\ -4x + y - 2z = 1 \\ 2x + 2y - z = 10 \end{cases}$$

eliminating z

① $x - 2y + z = -4$
③ $+\ 2x + 2y - z = 10$
$3x = 6$
$x = 2$

plug into ⑤

2· ① $2x - 4y + 2z = -8$
② $+\ -4x + y - 2z = 1$
$-2x - 3y = -7$

$-2(2) - 3y = -7$
$-4 - 3y = -7$
$-3y = -3$
$y = 1$

① $2 - 2(1) + z = 4$
$2 - 2 + z = 4$
$z = 4$

$(2, 1, 4)$

4. You manage a clothing store and budget $5400 to restock 200 shirts. You can buy T-shirts for $12 each, polo shirts for $24 each, and rugby shirts for $36 each. Suppose you want to have the same number of T-shirts as polo shirts. How many of each shirt should you buy?

① $12x + 24y + 36z = 5400$

② $x + y + z = 200$

③ $x = y$

replace all y's with x's

$12x + 24x + 36z = 5400$

④ $36x + 36z = 5400$

$x + x + z = 200$

⑤ $2x + z = 200$

④ $36x + 36z = 5400$

-18 ⑤ $-36x - 18z = -3600$

$18z = 1800$

$z = 100$

$2x + 100 = 200$

$2x = 100$

$x = 50$

$y = 50$

(50, 50, 100)

T polo rugby