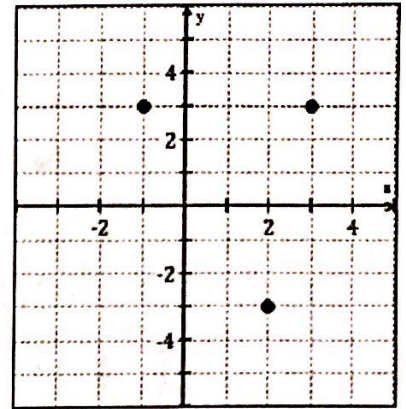


4.3 Modeling with Quadratic Functions

Three-Point Method

How could you write the equations for a parabola if you only knew three points on the curve and none of these were the vertex?



Writing the Equation of Parabola

1. create a system of eqv. by subbing given points
 2. for x, y in $ax^2 + bx + c = y$
 3. solve system for a, b, c
- plug in a, b, c

Ex 1. A parabola contains the points $(-1, 3)$, $(2, -3)$, and $(3, 3)$. Find the equation of this parabola in standard form.

$$ax^2 + bx + c = y$$

$$\left. \begin{aligned} a(-1)^2 + b(-1) + c &= 3 \\ a(2)^2 + b(2) + c &= -3 \\ a(3)^2 + b(3) + c &= 3 \end{aligned} \right\} \begin{aligned} 1a - 1b + c &= 3 \\ 4a + 2b + c &= -3 \\ 9a + 3b + c &= 3 \end{aligned}$$

Solve by hand or use matrix \rightarrow rref

$$\begin{array}{l} \textcircled{1} \\ \textcircled{2} \\ \textcircled{3} \end{array} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 3 \\ 4 & 2 & 1 & -3 \\ 9 & 3 & 1 & 3 \end{array} \right] \xrightarrow{\text{rref}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & -3 \end{array} \right] \begin{array}{l} a \\ b \\ c \end{array}$$

$$2x^2 - 4x - 3 = y$$

Ex 2. Find the equation for a parabola passing through (1,-2), (2,-4), and (3,-4).

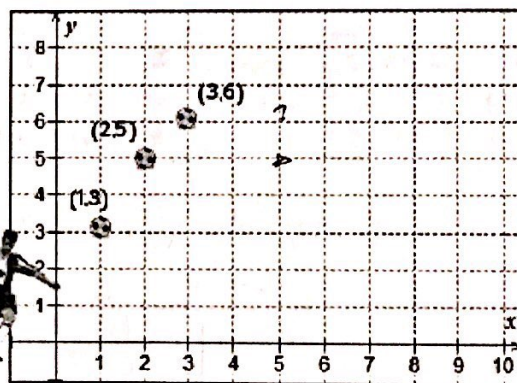
$$ax^2 + bx + c = y$$

$$\begin{cases} a(1^2) + b(1) + c = -2 \\ a(2^2) + b(2) + c = -4 \\ a(3^2) + b(3) + c = -4 \end{cases} \Rightarrow \begin{cases} 1a + 1b + c = -2 \\ 4a + 2b + c = -4 \\ 9a + 3b + c = -4 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & -2 \\ 4 & 2 & 1 & -4 \\ 9 & 3 & 1 & -4 \end{array} \right] \xrightarrow{\text{rref}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 2 \end{array} \right] \begin{matrix} a \\ b \\ c \end{matrix}$$

$$x^2 - 5x + 2 = y$$

Ex 3. The parabolic path of a soccer ball can be modeled by the graph at the right. The top of the goal cage is at (5,6). Will the ball go over the goal? If not, will it hit the goal? Explain.



$$\begin{cases} a(1)^2 + b(1) + c = 3 \\ a(2)^2 + b(2) + c = 5 \\ a(3)^2 + b(3) + c = 6 \end{cases} \Rightarrow \begin{cases} 1a + 1b + c = 3 \\ 4a + 2b + c = 5 \\ 9a + 3b + c = 6 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 4 & 2 & 1 & 5 \\ 9 & 3 & 1 & 6 \end{array} \right] \xrightarrow{\text{rref}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -0.5 \\ 0 & 1 & 0 & 3.5 \\ 0 & 0 & 1 & 0 \end{array} \right] \quad \begin{aligned} & -0.5x^2 + 3.5x = y \\ & \text{When } x=5 \dots y=5 \\ & -5(5^2) + 3.5(5) = 5 \end{aligned}$$

the ball is below the goal cage.