4.8 Complex Numbers

Main Ideas/Questions	Notes/Examples
The Imaginary Numbers	• Equations such as $x^2 + 1 = 0$ have no real solution, so mathematicians defined **imaginary numbers** to represent their solutions. • The **imaginary unit** i, is defined as $\sqrt{-1}$. This is useful when working with square roots of negative numbers. • A **pure imaginary number** is written in the form bi, where b is the real number and i is the imaginary part.

Simplifying Negative Square Roots

Step 1: rewrite $\sqrt{-a}$ as $\sqrt{-1}\sqrt{a}$

Step 2: simplify

Step 3: write as a pure imaginary

1. $\sqrt{-9}$ $\sqrt{-1}\sqrt{9}$ $i\sqrt{9}$ $\pm 3i$	2. $\sqrt{-196}$ $\sqrt{-1}\sqrt{196}$ $i\sqrt{196}$ $\pm 14i$	3. $\sqrt{-5}$ $\sqrt{-1\cdot 5}$ $i\sqrt{5}$ $\pm\sqrt{5}\,i$
4. $\sqrt{-80}$ $\sqrt{-1}\sqrt{80}$ $i\sqrt{16\cdot 5}$ $\pm 4\sqrt{5}\,i$	5. $\sqrt{-32}$ $\sqrt{-1}\sqrt{32}$ $i\sqrt{16\cdot 2}$ $\pm 4\sqrt{2}\,i$	6. $\sqrt{-192}$ $\sqrt{-1}\sqrt{192}$ $i\sqrt{64\cdot 3}$ $\pm 8\sqrt{3}i$

Solving Equations

7. $4x^2 + 15 = -9$

$4x^2 = -24$

$x^2 = -6$

$x = \pm\sqrt{-6}$

$x = \pm\sqrt{6}\,i$

8. $x^2 + 13 = 1$

$x^2 = -12$

$x = \pm\sqrt{-12}$

$x = \pm\sqrt{-1\cdot 4\cdot 3}$

$x = \pm 2\sqrt{3}\,i$

9. $5x^2 + 9 = 4x + 5$

$5x^2 - 4x + 4 = 0$

$\frac{4 \pm \sqrt{(-4)^2 - 4(5)(4)}}{2(5)}$

$\frac{4 \pm \sqrt{16 - 80}}{10}$

$\frac{4 \pm \sqrt{-64}}{10} = \frac{4 \pm 8i}{10} = \frac{2 \pm 4i}{5}$

10. $-3x^2 + 8x - 7 = 0$

$\frac{-8 \pm \sqrt{8^2 - 4(-3)(-7)}}{2(-3)}$

$\frac{-8 \pm \sqrt{64 - 84}}{-6}$

$\frac{-8 \pm \sqrt{-20}}{-6}$

$\frac{-8 \pm 2\sqrt{5}i}{-6} = \frac{-4 \pm \sqrt{5}i}{-3}$

Powers of "i"

$i^1 = \sqrt{-1} = i$

$i^2 = \sqrt{-1} \cdot \sqrt{-1} = -1$

$i^3 = i^2 \cdot i = -1\sqrt{-1} = -i$

$i^4 = i^2 \cdot i^2 = -1 \cdot -1 = 1$

$i^5 = i^4 \cdot i^1 = 1 \cdot i = i$

$i^6 = i^3 \cdot i^3 = -i(-i) = i^2 = -1$

$i^7 = i^4 \cdot i^3 = 1(-i) = -i$

$i^8 = i^4 \cdot i^4 = 1 \cdot 1 = 1$

11. i^{15}

$i^{12} \cdot i^3 = -i$

12. i^{62}

$i^{60} \cdot i^2 = -1$

Products of Pure Imaginary Numbers

Directions: Simplify the expressions below.

13. $4i \cdot 7i$

$28i^2$

$28(-1)$

-28

14. $(+4i)(2i)(+9i)$

$72i^3$

$72(-i)$

$-72i$

15. $(2i)^3 \cdot (5i)$

$8i^3 \cdot 5i$

$40i^4$

40

16. $(i\sqrt{3})^2 \cdot (-8i)^2$

$i^2(3) \cdot 64i^2$

$192i^4$

192

17. $\sqrt{-18} \cdot \sqrt{-10}$

$i\sqrt{18} \cdot i\sqrt{10}$

$i^2\sqrt{180} = i^2\sqrt{36 \cdot 5}$

$-1(6)\sqrt{5}$

$-6\sqrt{5}$

take out the "i"s first!

18. $\sqrt{-24} \cdot \sqrt{-3} \cdot \sqrt{-2}$

$i\sqrt{24}\ i\sqrt{3}\ i\sqrt{2}$

$i^3\sqrt{144}$

$12i^3 = 12(-i)$

$-12i$