Polynomials

- A **polynomial** is the sum or difference of many monomials.
- The **highest exponent** of a polynomial is called the degree
- **Standard Form:** Written with exponents in DECENDING ORDER

alphabetical

Write the polynomials below in standard form:

a's first

x's first

1. $-k^5 - 1 + 8k - 3k^3 + \frac{1}{4}k^2$ $\quad -k^5 - 3k^3 + \frac{1}{4}k^2 + 8k - 1$
2. $18a^2b^2 + 7ab - b^2 + 4a^3$ $\quad 4a^3 + 18a^2b^2 + 7ab - b^2$
3. $5xy^2 - x^2 + 9x^3y - y^4 + 2$ $\quad 9x^3y - x^2 + 5xy^2 - y^4 + 2$

Classifying Polynomials

Polynomials are **classified by degree** (highest exponent) and **number of terms.** Use the charts to the left to classify each polynomial below.

Degree	
0	constant
1	linear
2	quadratic
3	cubic
4	quartic
5	quintic

"n"th degree

Number of Terms	
1	monomial
2	binomial
3	trinomial
4+	polynomial

4. $-3x + 1$ $\quad$ linear binomial
5. $9x^5 - x^4 + 2x$ $\quad$ Quintic trinomial
6. 24 $\quad$ Constant monomial
7. $\frac{1}{2}x^3 - 2x^2 + 4x + 15$ $\quad$ Cubic polynomial
8. $-x^2 - 18x + 31$ $\quad$ Quadratic trinomial
9. $-\frac{3}{2}x^4$ $\quad$ Quartic Monomial
10. Give an example of a cubic binomial. $\quad x^3 + 7$
11. Given an example of a linear monomial. $\quad 4x$

Polynomial Functions

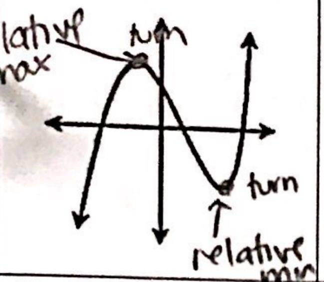
A polynomial function is a function of the form:

$$P(x) = a_nx^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_2x^2 + a_1x + a_0, \ldots$$

The coefficients $(a_n, a_{n-1}, \ldots a_1, a_0)$ are real numbers, $a_n \neq 0$ and n is a positive integer. (No negative exponents!)

a_n is called the leading coefficient

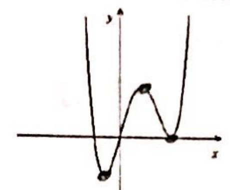
Turning Points



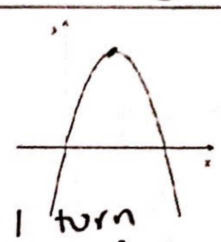
- The point(s) at which a polynomial function **switches direction.**
- If the turning points is **higher** than any nearby point, it is called a relative maximum
- If the turning point is **lower** than any nearby point, it is called a relatne minimum.
- Maximum and minimum values are all called extrema.
- at most $n-1$ turning points. n = degree

End Behavior Patterns

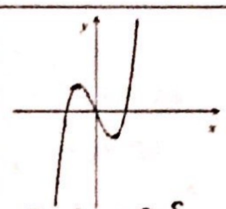
EVEN FUNCTIONS (degree) | ODD FUNCTIONS (degree)



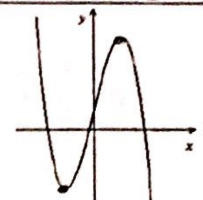
3 turns
$y = x^4 - 3x^3 + 4x$
up - up



1 turn
$y = -x^2 + 4x$
down - down



2 turns
$y = x^3 - 2x$
down - up



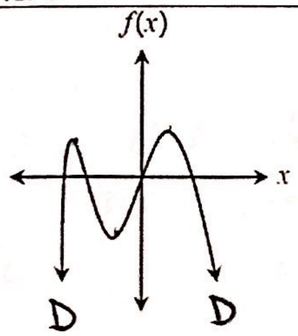
2 turns
$y = -0.5x^3 + 4x + 1$
up - down

	EVEN degree	**ODD** degree
POSITIVE Leading Coefficient	↑ ↗ up-up	↗ ↙ down-up
NEGATIVE Leading Coefficient	down-down ↙ ↓	↖ up-down ↘

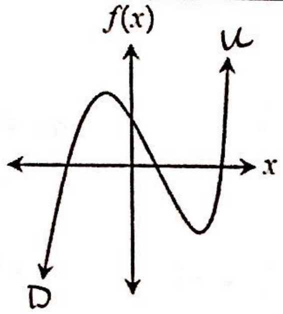
Determine the end behavior for the following functions.

12. $f(x) = 4x^4 + 6x^3 - x$ pos even up-up	13. $f(x) = -x^2 + x$ neg even down-down
14. $f(x) = 2x^3$ pos odd down-up	15. $f(x) = -x^3$ neg odd up-down
16. $-2x^2 + 4x - 7$ neg even down-down	17. $f(x) = \frac{1}{2}x^3 + 7$ pos odd down-up

Determine the sign of the leading coefficient and whether the function has an even or odd degree.



negative - even



positive - odd

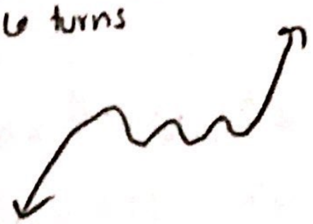
Consider the following equations. Sketch what the function **could** look like. (Make sure to include the correct end behavior and turning points)

$y = -4x^3 = 2x^2 + 7$

negative odd → U-D
2 turns

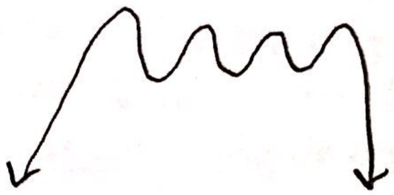
$y = 4x^7 + 2x - 1$

pos - odd → D-U
6 turns



$y = -2x^8 - 1$

negative even → D-D
7 turns



$y = 2x^4 + 3x^3 - 2x^2 + 7x - 9$

positive - even → U-U
3 turns

Using Differences to Determine Degree

Use the table below to determine the degree of the polynomial.

x	y
-3	23
-2	-16
-1	-15
0	-10
1	-13
2	-12
3	29

X- constant change

23 -16 -15 -10 -13 -12 29

-39 +1 +5 -3 +1 +41 1st

+40 +4 -8 +4 +40 2nd

-36 -12 +12 +36 3rd

+24 +24 +24 4th

Since the 4^{th} differences are the same, the table represents a 4^{th}-degree (quartic) polynomial