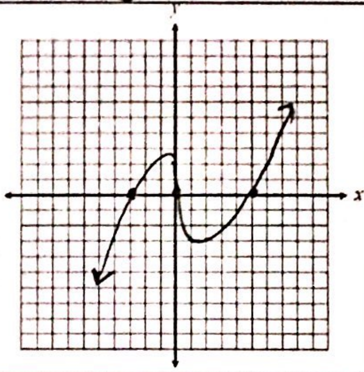
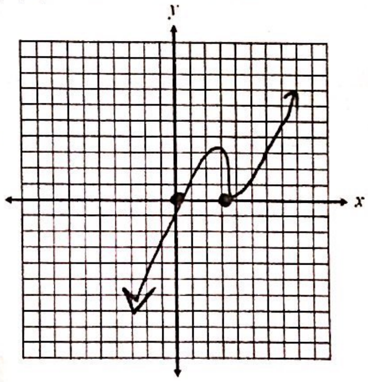
5.2 Polynomial Linear Factors and Zeros

ZEROS solutions x-intercepts roots	• The point(s) at which a function intersects the x-axis • A polynomial function of degree n can have at most n zeros • An **EVEN** polynomial has an even number of zeros (or no zeros). • An **ODD** polynomial has an odd number of zeros.
LINEAR FACTORS	The **linear factors** of a polynomial reveal its zeros. Linear factors can be found by writing a polynomial in **factored form.** $(x \pm a)(x \pm b)(x \pm c)$
	1. Write $f(x) = x^3 - 2x^2 - 15x$ in factored form. pos, odd D-U $f(x) = x(x^2 - 2x - 15)$ $f(x) = x(x-5)(x+3)$ What are the zeros? 0, 5, -3 Sketch the graph to the right. 
MULTIPLICITY	The **multiplicity** of a zero is equal to the number of times its corresponding linear factor occurs.
	2. Write $f(x) = x^3 - 6x^2 + 9x$ in factored form. $f(x) = x(x^2 - 6x + 9)$ $f(x) = x(x-3)(x-3)$ $\quad$ $f(x) = x(x-3)^2$ What are the zeros and their multiplicity? 0 (multiplicity of 1), 3 (multiplicity of 2)
How MULTIPLICITY affects a GRAPH	3. Find the zeros of the following function and graph. $f(x) = x^3 - 6x^2 + 9x$ $x(x^2 - 6x + 9)$ $\quad$ D-U $x(x-3)^2$ $0^1, 3^2$ ↓ cross $\quad$ ↓ bounce  • If the **multiplicity** is **ODD**, then the graph crosses at that zero. • If the **multiplicity** is **EVEN**, then the graph bounces at that zero.

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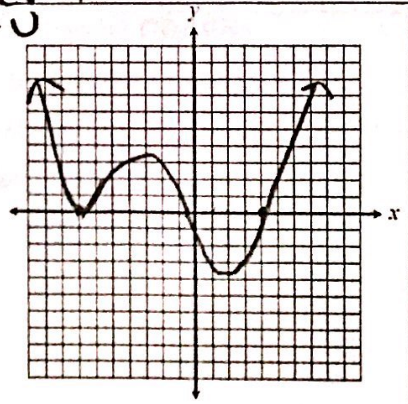
Examples	
Identify the zeros and state their multiplicities. Describe the effect on the graph. Graph the function.	4. $f(x) = (x+7)^2(2x+1)(x-4)^3$

$2 + 1 + 3 = 6$

pos, even

U-U

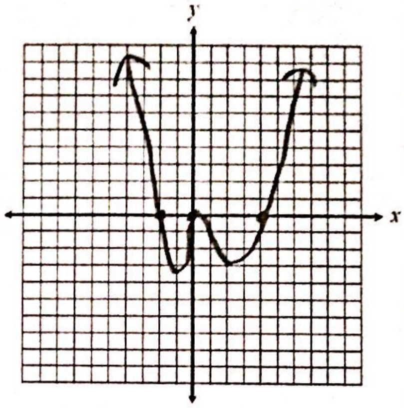


$x = -7, -\frac{1}{2}, 4$

Zero	Multiplicity	Effect
-7	2	bounce
-1/2	1	cross
4	3	cross

5. $f(x) = x^4 - 2x^3 - 8x^2$

pos-even



$x^2(x^2 - 2x - 8)$

$x^2(x-4)(x+2)$

$0, 4, -2$

Zero	Multiplicity	Effect
0	2	bounce
4	1	cross
-2	1	cross

Writing a Polynomial from its Zeros

6. What is a cubic polynomial in standard form with zeros at -2, 3, and 5?

$x = -2 \quad x = 3 \quad x = 5$

$(x+2) \quad (x-3) \quad (x-5)$

$(x+2)(x-3)(x-5)$

$(x^2 - x - 6)(x-5)$

$x^3 - 5x^2 - x^2 + 5x - 6x + 30$

$f(x) = x^3 - 6x - x + 30$

7. A polynomial function has a zero at -6 (multiplicity of 2) and 2 (multiplicity of 1). Write a polynomial function in standard form that could represent the function.

$x = -6 \quad x = -6 \quad x = 2$

$(x+6)(x+6)(x-2)$

$(x^2 + 12x + 36)(x-2)$

$x^3 - 2x^2 + 12x^2 - 24x + 36x - 72$

$f(x) = x^3 + 10x^2 + 12x - 72$

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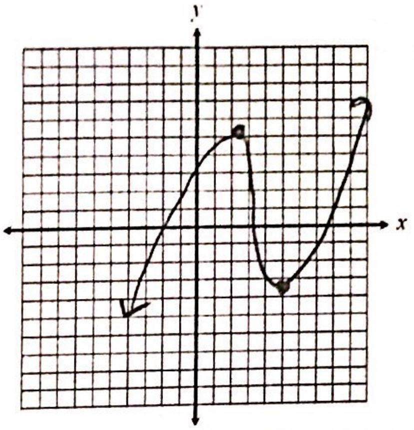
Using a calculator to find the turning points	**Step 1:** Graph the function. (Enter the function in **y=**, then hit **GRAPH**) **Step 2:** Use the **CALC** menu to find the minimum and maximum values. **Step 3:** Move the cursor to the left bound of the turning point. Hit **ENTER**, then move the cursor to the right bound of the turning point. Hit **ENTER**, and guess where the max/min is. Hit **ENTER**. 8. Find the turning points of the function $f(x) = x^3 - 6x^2 + 7x + 2$ Sketch the graph to the right.  Rel. Maximum(s): (0.709, 4.303) Rel. Minimum(s): (3.291, -4.303)

9. The design of a digital box camera maximizes the volume while keeping the sum of the dimensions at 6 inches. If the length must be 1.5 times the height, what should each dimension be?

a. Define a variable x.

x = height

b. Determine the length and width in terms of x.

length = 1.5x width = 6-(x)-1.5x =6-2.5x

c. Model the volume.

V=LWH

V=(1.5x)(6-2.5x)(x) = -3.75x³+9x²

d. Graph the polynomial function and use the MAXIMUM feature to find the maximum volume and where that occurs.

x=1.6

max @ (1.6, 7.68)

e. Find the dimensions of the camera.

height= 1.6 in

length= 1.5(1.6) = 2.4 in

width= 6-2.4-1.6= 2 in

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