5.3 Solving Polynomial Equations

Find the real or imaginary solutions of each equation by factoring

1. $x^3 + 27 = 0$	2. $3x^3 - 24 = 0$

1. $x^3 + 27 = 0$

$(x+3)(x^2-3x+9)$

$x+3=0$ $\quad$ $\dfrac{3 \pm \sqrt{(-3)^2-4(1)(9)}}{2}$

$x=-3$ $\quad$ $\dfrac{3 \pm \sqrt{-27}}{2}$

$\dfrac{3 \pm 3\sqrt{3}\,i}{2}$

$x = -3, \dfrac{3}{2} \pm \dfrac{3\sqrt{3}}{2}i$

2. $3x^3 - 24 = 0$

$3(x^3-8)=0$

$3(x-2)(x^2+2x+4)$ $\quad$ complete sq.

$x-2=0$ $\qquad$ or quad. formula

$x=2$ $\qquad x^2+2x+4=0$

$\qquad\qquad x^2+2x=-4$

$\qquad\qquad x^2+2x+1=-3$

$\qquad\qquad (x+1)^2=-3$

$\qquad\qquad x+1=\pm\sqrt{-3}$

$x=2, -1\pm\sqrt{3}i \qquad x=-1\pm\sqrt{3}i$

3. $6x^2 + 13x = 5$ $\quad 6x^2+13x-5=0 \quad ac=-30$

$6x^2+15x-2x-5=0 \qquad 15 \quad -2$

$3x(2x+5)-1(2x+5)=0$

$(3x-1)(2x+5)=0$

$x = 1/3, -5/2$

4. $x^3 + 2x^2 + 9x + 18 = 0$

$x^2(x+2)+9(x+2)$

$(x^2+9)(x+2)=0$

$x^2+9=0 \qquad x=-2$

$x^2=-9$

$x=\pm\sqrt{-9}$

$x=\pm 3i$

$x=-2, \pm 3i$

5. $x^4 + 2x^2 - 15 = 0$

$(x^2+5)(x^2-3)=0$

$x^2+5=0 \qquad x^2-3=0$

$x^2=-5 \qquad\quad x^2=3$

$x=\pm\sqrt{-5} \qquad x=\pm\sqrt{3}$

$x=\pm\sqrt{5}i$

$x=\pm\sqrt{5}i, \pm\sqrt{3}$

6. $x^3 + 3x^2 + 6x = 0$

$x(x^2+3x+6)=0$

$x=0 \qquad \dfrac{-3\pm\sqrt{3^2-4(1)(6)}}{2}$

$\qquad\qquad \dfrac{-3\pm\sqrt{9-24}}{2}$

$\qquad\qquad \dfrac{-3\pm\sqrt{-15}}{2} = \dfrac{-3\pm\sqrt{15}\,i}{2}$

$x=0, \dfrac{-3\pm\sqrt{15}}{2}i$

7. The volume V of a shipping container is 240 ft³. The width, the length, and the height are x, $(x + 2)$, and $(x - 5)$ respectively. What are the dimensions of the container?

$V = LWH$
$240 = x(x+2)(x-5)$
$240 = x(x^2 - 3x - 10)$
$240 = x^3 - 3x^2 - 10x$

$x^3 - 3x^2 - 10x - 240 = 0$

calculator

$x = 8$

8 by 10 by 3

5.4 Dividing Polynomials

Divide using long division.

8. $(x^2 - 5x - 36) \div (x + 4)$

$$\begin{array}{r} x - 9 \\ x+4\overline{)x^2 - 5x - 36} \\ -(x^2 + 4x) \downarrow \\ \hline -9x - 36 \\ -(-9x - 36) \\ \hline 0 \end{array}$$

$x + 9$

9. $(3x^3 - x^2 + 7x - 6) \div (x - 2)$

$$\begin{array}{r} 3x^2 + 5x + 17 \\ x-2\overline{)3x^3 - x^2 + 7x - 6} \\ -(3x^3 - 6x^2) \downarrow \\ \hline 5x^2 + 7x \\ -(5x^2 - 10x) \downarrow \\ \hline 17x - 6 \\ -(17x - 34) \\ \hline 28 \end{array}$$

$3x^2 + 5x + 17$ R 28

Divide using synthetic division.

10. $(x^3 - x + 6) \div (x + 2)$ $x = -2$

$$\begin{array}{r|rrrr} -2 & 1 & 0 & -1 & 6 \\ + & \downarrow & -2 & 4 & -6 \\ \hline & 1 & -2 & 3 & \boxed{0} \end{array}$$

$x^2 - 2x + 3$

11. $(x^4 - 3x^2 + 7x - 18) \div (x - 2)$ $x = 2$

$$\begin{array}{r|rrrrr} 2 & 1 & 0 & -3 & 7 & -18 \\ + & \downarrow & 2 & 4 & 2 & 18 \\ \hline & 1 & 2 & 1 & 9 & \boxed{0} \end{array}$$

$x^3 + 2x^2 + x + 9$

Use synthetic division and the given factor to completely factor the polynomial.

12. $(x^3 - 3x^2 - 10x + 24) : (x - 2)$

$$\begin{array}{r|rrrr} 2 & 1 & -3 & -10 & 24 \\ + & \downarrow & 2 & -2 & -24 \\ \hline & 1 & -1 & -12 & \boxed{0} \end{array}$$

$x^2 - x - 12$

$(x-4)(x+3)(x-2)$

13. $(x^3 + 4x^2 - 11x - 30) : (x + 5)$

$$\begin{array}{r|rrrr} -5 & 1 & 4 & -11 & -30 \\ + & \downarrow & -5 & 5 & 30 \\ \hline & 1 & -1 & -6 & \boxed{0} \end{array}$$

$x^2 - x - 6$

$(x-3)(x+2)(x+5)$

Use synthetic division and the Remainder Theorem to find P(a).

14. $x^3 + 5x^2 - 12x + 3 ; a = -4$

$$
\begin{array}{r|rrrr}
-4 & 1 & 5 & -12 & 3 \\
  &   & -4 & -4 & 64 \\
\hline
  & 1 & 1 & -16 & 67
\end{array}
$$

$P(-4) = 67$

15. $2x^3 - 4x^2 - 5x + 7 ; a = 2$

$$
\begin{array}{r|rrrr}
2 & 2 & -4 & -5 & 7 \\
  &   & 4 & 0 & -10 \\
\hline
  & 2 & 0 & -5 & -3
\end{array}
$$

$P(2) = -3$

5.5 Theorems About Roots of Polynomials

Use the Rational Roots Theorem to list all possible rational roots for each equation. Then find any rational roots.

16. $x^3 + 6x^2 - 7x - 12 = 0$

$p \to 1, 2, 3, 4, 6, 12$

$q \to 1$

$P/q \to \pm1, \pm2, \pm3, \pm4, \pm6, \pm12$

$x = -1$ rational

$$
\begin{array}{r|rrrr}
-1 & 1 & 6 & -7 & -12 \\
   &   & -1 & -5 & 12 \\
\hline
   & 1 & 5 & -12 & 0
\end{array}
$$

$x^2 + 5x - 12$

17. $x^4 + x^2 - 20 = 0$

$p \to 1, 2, 4, 5, 10, 20$

$q \to 1$

$p/q = \pm\{1, 2, 4, 5, 10, 20\}$

$$
\begin{array}{r|rrrrr}
2 & 1 & 0 & 1 & 0 & -20 \\
  &   & 2 & 4 & 10 & 20 \\
\hline
  & 1 & 2 & 5 & 10 & 0
\end{array}
$$

$x^3 + 2x^2 + 5x + 10$

$x^3 + 2x^2 + 5x + 10$

$x^2(x+2) + 5(x+2)$

$(x^2 + 5)(x + 2)$

$x = -2$

$x = 2, -2$ rational

Write a polynomial function with rational coefficients so that P(x)=0 has the given roots.

18. 7 and -3

$(x-7)(x+3)$

$x^2 - 4x - 21$

19. $\sqrt{5}$ and $3i$

$(x+\sqrt{5})(x-\sqrt{5})(x+3i)(x-3i)$

$(x^2 + \sqrt{5}x - \sqrt{5}x - 5)(x^2 + 3ix - 3ix - 9i^2)$

$(x^2 - 5)(x^2 + 9)$

$x^4 + 4x^2 - 45$

20. $3 \ \& \ \sqrt{6}$

$(x-3)(x+\sqrt{6})(x-\sqrt{6})$

$(x-3)(x^2 + \sqrt{6}x - \sqrt{6}x - 6)$

$(x-3)(x^2 - 6)$

$x^3 - 3x^2 - 6x + 18$

21. $2 \ \& \ 6i$

$(x-2)(x+6i)(x-6i)$

$(x-2)(x^2 + 6ix - 6ix - 36i^2)$

$(x-2)(x^2 + 36)$

$x^3 - 2x^2 + 36x - 72$