

5.3 Solving Polynomials

Key

Always check for a GCF first!

DIFFERENCE OF SQUARES $a^2 - b^2$	Perfect Squares: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144
	Rule: $a^2 - b^2 = (a+b)(a-b)$
	1. $4x^4 - 9$ $(2x^2 + 3)(2x^2 - 3)$
	2. $10w^5 - 10w$ $10w(w^4 - 1)$ $10w(w^2 + 1)(w^2 - 1)$ $10w(w^2 + 1)(w + 1)(w - 1)$
QUADRATIC TRINOMIALS	3. $6x^2 - 5x - 4$ $6x^2 - 8x + 3x - 4$ $AC = -24$ $-8, 3$ $2x(3x - 4) + 1(3x - 4)$ $(2x + 1)(3x - 4)$
	4. $x^2 + x - 42$ $(x + 7)(x - 6)$
QUADRATIC-LIKE TRINOMIALS $ax^4 + bx^2 + c$	5. $x^4 + 2x^2 - 24$ $(x^2 + 6)(x^2 - 4)$ $(x^2 + 6)(x + 2)(x - 2)$
	6. $3x^4 + 14x^2 - 5$ $3x^4 + 15x^2 - x^2 - 5$ $AC = -15$ $15, -1$ $3x^2(x^2 + 5) - 1(x^2 + 5)$ $(3x^2 - 1)(x^2 + 5)$
SUM OF CUBES $a^3 + b^3$ SOAP $\begin{matrix} a & \cancel{b} \\ \cancel{m} & \cancel{e} \end{matrix}$	Perfect Cubes: 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000
	Rule: $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
	7. $x^3 + 8$ $a = x \quad b = 2$ $(x + 2)(x^2 - 2x + 4)$
	8. $64x^3 + 1$ $a = 4x \quad b = 1$ $(4x + 1)(16x^2 - 4x + 1)$

DIFFERENCE OF CUBES $a^3 - b^3$ SOAP	Rule: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$	
	9. $x^3 - 27$ $a = x \quad b = 3$ $(x - 3)(x^2 + 3x + 9)$	10. $125x^3 - 1$ $a = 5x \quad b = 1$ $(5x - 1)(25x^2 + 25x + 1)$
4 TERMS grouping	11. $x^3 - 4x^2 - 9x + 36$ $x^2(x - 4) - 9(x - 4)$ $(x^2 - 9)(x - 4)$ $(x + 3)(x - 3)(x - 4)$	12. $2x^5 - 18x^4 + 7x - 63$ $2x^4(x - 9) + 7(x - 9)$ $(2x^4 + 7)(x - 9)$
Solve the following by factoring. Simplify all irrational and complex solutions.		
13. $(x^2 - 1)(x^2 + 4) = 0$ $(x+1)(x-1)(x^2+4) = 0$ $x+1=0 \quad x-1=0 \quad x^2+4=0$ $x=-1 \quad x=1 \quad x^2=-4$ $x=\pm\sqrt{-4}$ $x=\pm 2i$ $x = \pm 1, \pm 2i$	14. $x^5 + 4x^3 = 5x^4 - 2x^3$ $x^5 - 5x^4 + 6x^3 = 0$ $x^3(x^2 - 5x + 6) = 0$ $x^3(x-2)(x-3) = 0$ $x^3=0 \quad x-2=0 \quad x-3=0$ $x=0(\text{mult 3}), 3, 2$	
15. $x^4 = 16$ $x^4 - 16 = 0$ $(x^2 - 4)(x^2 + 4) = 0$ $(x+2)(x-2)(x^2+4) = 0$ $x+2=0 \quad x-2=0 \quad x^2+4=0$ $x=-2 \quad x=2 \quad x^2=-4$ $x=\pm 2i$ $x = -2, 2, \pm 2i$	16. $x^3 = 8x - 2x^2$ $x^3 + 2x^2 - 8x = 0$ $x(x^2 + 2x - 8) = 0$ $x(x+4)(x-2) = 0$ $x=0, -4, 2$	

Quad-like trinomial

17. $x(x^2 + 8) = 8(x+1)$

$$x^3 + 8x = 8x + 8$$

$$x^3 - 8 = 0$$

$$(x-2)(x^2 + 2x + 4) = 0$$

$$x-2=0$$

$$x^2 + 2x + 4 = 0$$

$$x^2 + 2x = -4$$

$$x^2 + 2x + 1 = -4 + 1$$

$$(x+1)^2 = -3$$

$$x+1 = \pm\sqrt{-3}$$

$$x+1 = \pm\sqrt{3}i$$

$$x = -1 \pm \sqrt{3}i$$

SOLVE
quad form
comp 1. sq

18. $x^4 - 9x^2 + 14 = 0$

$$(x^2 - 7)(x^2 - 2) = 0$$

$$x^2 = 2$$

$$x = \pm\sqrt{2}$$

$$x^2 = 7$$

$$x = \pm\sqrt{7}$$

$$x = \pm\sqrt{7}, \pm\sqrt{2}$$

19. $x^4 + 6x^2 = 27$

$$x^4 + 6x^2 - 27 = 0$$

$$(x^2 + 9)(x^2 - 3) = 0$$

$$x^2 + 9 = 0 \quad x^2 - 3 = 0$$

$$x^2 = -9 \quad x^2 = 3$$

$$x = \pm\sqrt{-9} \quad x = \pm\sqrt{3}$$

$$x = \pm 3i$$

$$x = \pm 3i, \pm\sqrt{3}$$

grouping

$$x^2(x+2) + 9(x+2) = 0$$

$$(x^2 + 9)(x+2) = 0$$

$$x^2 + 9 = 0 \quad x+2 = 0$$

$$x^2 = -9 \quad x = -2$$

$$x = \pm\sqrt{9}$$

$$x = \pm 3i$$

$$x = -2, \pm 3i$$

21. Find three consecutive integers whose product is 480 more than their sum.

$$x, x+1, x+2$$

$$[7, 8, 9]$$

$$(x)(x+1)(x+2) = x + x+1 + x+2 + 480$$

$$x(x^2 + 3x + 2) = 3x + 483$$

$$x^3 + 3x^2 + 2x = 3x + 483$$

$$x^3 + 3x^2 - x - 483 = 0$$

SOLVE USING a
calculator!

$$x = 7$$

22. The Perez twins were born two years after their older sister. This year, the product of the three sibling's ages is exactly 4558 more than the sum of their ages. How old are the twins?

twins $\rightarrow x, x$

sister $\rightarrow x+2$

$$x(x)(x+2) = x + x + x + 2 + 4558$$

$$x^2(x+2) = 3x + 4560$$

$$x^3 + 2x^2 - 3x - 4560 = 0$$

SOLVE USING a
calculator!

$$x = 16 \text{ yrs old}$$