5.4 Dividing Polynomials

Numerical Long Division	**1.** Solve using long division. 672 ÷ 21 32 $21\overline{)672}$ divisor ↗ $-63\downarrow$ ↖ dividend 42 -42 0 $672 \div 21 = 32$	**2.** Solve using long division. 5783 ÷ 3 $5783 \div 3 =$ 1927 $3\overline{)5783}$ -3 2783 27 83 6 23 21 2 $1927\ R2$ $1927\frac{2}{3}$
Polynomial Long Division	**3.** Divide. What is the quotient and remainder? $(4x^2+9x+6)\div(x+6)$ $4x-15$ $x+6\overline{)4x^2+9x+6}$ $-\cancel{4x^2}+24x\downarrow$ $-15x+6$ $-\ \cancel{-15x}-90$ 96 $4x^2+9x+6 \div x+6 = 4x-15\ R96$ $\boxed{4x-15+\frac{96}{x+6}}$	**4.** Divide. What is the quotient and remainder? $(3x^2-29x+56)\div(x-7)$ $3x-8$ $x-7\overline{)3x^2-29x+56}$ $-\ \cancel{3x^2}-21x\downarrow$ $-8x+56$ $-\ \cancel{-8x}+\cancel{56}$ 0 $\boxed{3x-8}$
*use place holders for missing terms Checking Factors • factors divide evenly with no remainder	**5.** Is $(x-2)$ a factor of x^3-9? x^2+2x+4 $x-2\overline{)x^3+0x^2+0x-9}$ $-\ \cancel{x^3}-2x^2\downarrow$ $\cancel{2x^2}+0x$ $-\ \cancel{2x^2}-4x$ ↓ $\cancel{4x}-9$ $-\ \cancel{4x}-8$ -1 x-2 is not a factor	**6.** Is $(x-4)$ a factor of $P(x)=5x^2-17x-12$? If it is, write $P(x)$ as a product of two factors. $5x+3$ $x-4\overline{)5x^2-17x-12}$ $-\ \cancel{5x^2}-20x\downarrow$ $3\cancel{x}-1\cancel{2}$ $-\ 3\cancel{x}-1\cancel{2}$ 0 yes, x-4 is a factor $P(x)=(x-4)(5x+3)$

Synthetic Division

1. Write the polynomial in standard form. Include zeros for any missing powers of x.
2. Omit all variables and exponents.
3. For the divisor, reverse the sign and use *a*. (Solve divisor for x, the answer is your "box" number)
4. Add instead of subtract throughout.

7. Divide $x^3 \overset{x^2}{-} 57x + 56$ by $x - 7$. What is the quotient and remainder? $x = 7$

$7 \,|\; 1 \quad 0 \quad -57 \quad 56$

$+ \quad \downarrow \quad 7 \quad 49 \quad -56$

$1 \quad 7 \quad -8 \quad \boxed{0}$

$x^2 \quad x \quad \#$

$Q \Rightarrow x^2 + 7x - 8$

$R \Rightarrow 0$

8. Divide $x^3 - 14x^2 + 51x - 54$ by $x + 2$. What is the quotient and remainder? $x = -2$

$-2 \,|\; 1 \quad -14 \quad 51 \quad -54$

$+ \quad \downarrow \quad -2 \quad 32 \quad -166$

$1 \quad -16 \quad 83 \quad \boxed{-220}$

$x^2 - 16x + 83 \quad R\ -220$

$x^2 - 16x + 83 - \frac{220}{x+2}$

9. If the polynomial $x^3 + 6x^2 + 11x + 6$ expresses the volume, in cubic inches, of a box, and the width is $(x + 1)$ inches, what are the dimensions of the box?

$-1 \,|\; 1 \quad 6 \quad 11 \quad 6$

$\downarrow \quad -1 \quad -5 \quad -6$

$1 \quad +5 \quad 6 \quad \boxed{0}$

$(x+1)(x^2+5x+6)$

$(x+1)(x+3)(x+2)$

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The Remainder Theorem

The Remainder Theorem provides a quick way to find the remainder of a polynomial long-division problem.

If you divide a polynomial $P(x)$ of degree $n \geq 1$ by $x - a$, then the remainder is $P(a)$

10. Find $P(-4)$, given that $P(x) = x^5 - 3x^4 - 28x^3 + 5x + 20$

box #

$-4 \,|\; 1 \quad -3 \quad -28 \quad 0 \quad 5 \quad 20$

$\downarrow \quad -4 \quad 28 \quad 0 \quad 0 \quad -20$

$1 \quad -7 \quad 0 \quad 0 \quad 5 \quad \boxed{0}$

$P(-4) = 0$

11. Determine if $(x - 3)$ is a factor of $x^4 - 81$.

$3 \,|\; 1 \quad 0 \quad 0 \quad 0 \quad -81$

$\downarrow \quad 3 \quad 9 \quad 27 \quad 81$

$1 \quad 3 \quad 9 \quad 27 \quad \boxed{0}$

yes, x-3 is a factor