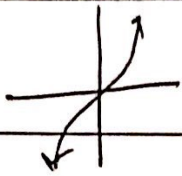
5.9 Transforming Polynomial Functions

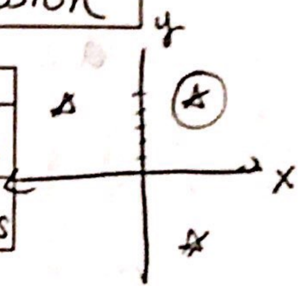
Essential Understanding: The graph of the function $y = af(x-h) + k$ is a vertical stretch or compression by the factor $|a|$, a horizontal shift of h units, and a vertical shift of k units of the graph $y = f(x)$.

Translations	
$f(x+h)$	left h
$f(x-h)$	right h
$f(x)+k$	up K
$f(x)-k$	down K

Dilations		
$a \cdot f(x)$	When $\lvert a\rvert > 1$	Stretch (by factor of a)
$a \cdot f(x)$	When $\lvert a\rvert < 1$ ($0<a<1$)	Compression



Reflections	
$-f(x)$	reflect x-axis
$f(-x)$	reflect y-axis



Transforming $y = x^3$ $y = a(x-h)^3 + K$

Ex. 1. What is an equation of the graph $y = x^3$ under a vertical compression by the factor ½ followed by a reflection across the x-axis, a horizontal translation 3 units to the right, and then a vertical translation 2 units up?

$-\frac{1}{2}(x-3)^3 + 2 = y$

Ex. 2. What is an equation of the graph $y = x^3$ under a vertical stretch by the factor 2 followed by a horizontal translation 3 units to the left and then a vertical translation 4 units down?

$y = 2(x+3)^3 - 4$

Finding Zeros of a Transformed Cubic Function

Ex. 3. What are the real zeros of the function $y = 3(x-1)^3 + 6$?

$3(x-1)^3 + 6 = 0$

$\frac{3(x-1)^3}{3} = \frac{-6}{3}$

$\sqrt[3]{(x-1)^3} = \sqrt[3]{-2}$

$x - 1 = \sqrt[3]{-2}$

$\boxed{x = 1 + \sqrt[3]{-2}}$

Ex. 4. What are the real zeros of the function $y = -3(x-1)^3 + 8$?

$-3(x-1)^3 + 8 = 0$

$-3(x-1)^3 = -8$

$(x-1)^3 = 8/3$

$x - 1 = \sqrt[3]{8/3}$

$x = 1 + \sqrt[3]{8/3}$

Ex. 5. What are the real zeros of the function $y = a(x-h)^3 + k$?

$a(x-h)^3 + K = 0$

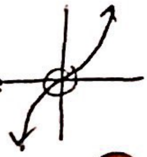
$a(x-h)^3 = -K$

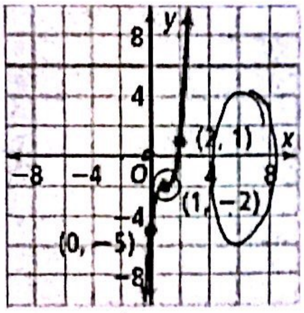
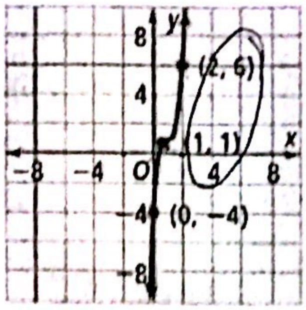
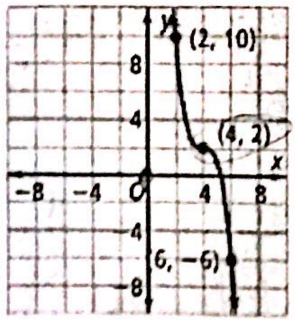
$(x-h)^3 = -K/a$

$x - h = \sqrt[3]{-K/a}$

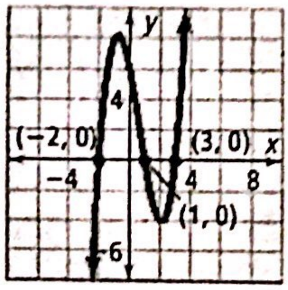
$x = h + \sqrt[3]{\frac{-K}{a}}$

Determine the transformations that were used to change the graph of the parent function $y = x^3$ to each of the following graphs.



		
• down 2 • right 1 • stretch by 3	• up 1 • right 1 • stretch by 5	• right 4 • reflect • up 2 • stretch of 4

The previous examples together illustrate that the graph of an "offspring" function of the parent cubic function $y = x^3$ has only one x-intercept.



The graph of the cubic function $y = x^3 - 2x^2 - 5x + 6$ has three x-intercepts (as shown in the figure to the right). You cannot obtain this function or others like it by transforming the parent cubic function $y = x^3$ using stretches, reflections, and translations.

Similarly, some quartic functions are simple transformations of $y = x^4$ and some are not.

Constructing a Quartic Functions with Two Real Zeros

What is a quartic function with only two real zeros, x=5 and x=9?

Method 1: Use Transformations

Method 2: Use Algebra

$X=5 \quad X=9 \quad | \quad X=i \quad X=-i$

$(x-5)(x-9) \quad | \quad (x-i)(x+i) \leftarrow -(-1)$

$x^2-5x-9x+45 \quad | \quad x^2 - \cancel{ix} + \cancel{ix} - i^2$

↓

$(x^2-14x+45) \quad (x^2+1)$

$x^4+x^2-14x^3-14x+45x^2+45$

$\boxed{x^4-14x^3+46x^2-14x+45}$

What is a quartic function f(x) with only two real zeros, x=0 and x=6? Will the function –f(x) have the same zeros?

$X=0 \quad x=6 \quad x=i \quad x=-i$

$(x)(x-6) \,|\, (x-i)(x+i)$

$(x^2-6x)(x^2+1)$

$\boxed{x^4-6x^3+x^2-6x}$