

4 Rational Exponents

Rational Exponents numerator \leftrightarrow index denominator \leftrightarrow exponent	Expressions with rational exponents can be rewritten as radicals using the following rules:		
	Exponential Form	Meaning	Radical Form
	$a^{\frac{1}{n}}$	The n^{th} root of a	$a^{\frac{1}{n}} = \sqrt[n]{a}$
	$a^{\frac{m}{n}}$	The n^{th} root of a , raised to the m^{th} power	$a^{\frac{m}{n}} = \sqrt[n]{a^m}$
Exponent Rules Review	$x^m \cdot x^n = x^{m+n}$	$\frac{x^m}{x^n} = x^{m-n}$	$(x^m)^n = x^{m \cdot n}$
			$x^{-m} = \frac{1}{x^m}$ $\frac{1}{x^2} = x^{-2}$

Converting between Exponential & Radical Form	Directions: Write each expression in radical form . Simplify if needed.		
	1. $x^{\frac{1}{4}}$ $\sqrt[4]{x}$	2. $(15n)^{\frac{1}{2}}$ $\sqrt{15n}$	3. $24^{\frac{1}{3}}$ $\sqrt[3]{24}$ $\sqrt[3]{8} \sqrt[3]{3}$ $2 \sqrt[3]{3}$
	4. $7^{\frac{2}{3}}$ $\sqrt[3]{7^2}$ $\sqrt[3]{49}$	5. $k^{\frac{7}{2}}$ $\sqrt{k^7}$ $k^3 \sqrt{k}$	6. $3^{\frac{5}{4}}$ $\sqrt[4]{3^5}$ $3 \sqrt[4]{3}$
	Directions: Write each expression in exponential form .		
	7. $\sqrt[3]{16}$ $\sqrt[3]{2^4}$ $2 \cdot 2^{\frac{1}{3}}$	8. \sqrt{xy} $(xy)^{\frac{1}{2}}$ $x^{\frac{1}{2}} y^{\frac{1}{2}}$	9. $\sqrt[4]{8w}$ $(8w)^{\frac{1}{4}}$ $8^{\frac{1}{4}} w^{\frac{1}{4}}$
	10. $\sqrt[3]{11^2}$ $11^{\frac{2}{3}}$ $121^{\frac{1}{3}}$	11. $(\sqrt{-2a})^5$ $(-2a)^{\frac{5}{4}}$ $\sqrt[4]{-32a^5}$ $\sqrt[4]{16a^4} \sqrt{-2a}$ $2a \sqrt{-2a}$ $2a(-2a)^{\frac{1}{4}}$	12. $\sqrt[3]{9x^7y^4}$ $(9x^7y^4)^{\frac{1}{3}}$ $9^{\frac{1}{3}} x^{\frac{7}{3}} y^{\frac{4}{3}}$

Simplifying Expressions with Rational Exponents

- 1 Rewrite all radicals in **exponential form**.
- 2 Use the **exponent rules** to simplify the expression.
- 3 Write answer as a radical in **simplest form**. Rationalize.

13. $x^{\frac{1}{3}} \cdot x^{\frac{4}{3}}$
 $x^{\frac{5}{3}}$
 $x^{\frac{3}{3}} \cdot x^{\frac{2}{3}}$
 $x \cdot x$
 $\sqrt[3]{x^5}$
 $x \sqrt[3]{x^2}$

14. $\frac{m^{\frac{5}{2}}}{m^{\frac{1}{4}}}$
 $\frac{5}{2} - \frac{1}{4}$
 $\frac{10}{4} - \frac{1}{4} = \frac{9}{4}$
 $m^{\frac{9}{4}}$
 $\sqrt[4]{m^9}$

15. $(a^{\frac{1}{3}})^{\frac{5}{2}}$
 $\frac{1}{3} \cdot \frac{5}{2} = \frac{5}{6}$
 $a^{\frac{5}{6}}$
 $\sqrt[6]{a^5}$

16. $(8x^2)^{\frac{2}{3}}$
 $\sqrt[3]{(8x^2)^2}$
 $\sqrt[3]{8^2 x^4}$
 $\sqrt[3]{64 x^4}$
 $4x \sqrt[3]{x}$

17. $(8x^{15})^{-\frac{1}{3}}$
 $\frac{1}{(8x^{15})^{\frac{1}{3}}}$
 $\frac{1}{\sqrt[3]{8x^{15}}} = \frac{1}{2x^5}$

18. $\sqrt[3]{v} \cdot \sqrt{v}$
 $v^{\frac{1}{3}} \cdot v^{\frac{1}{2}}$
 $\frac{1}{3} + \frac{1}{2} = \frac{2}{6} + \frac{3}{6} = \frac{5}{6}$
 $v^{\frac{5}{6}} = \sqrt[6]{v^5}$

19. $\frac{4}{\sqrt[3]{4}}$
 $1 - \frac{1}{3} = \frac{2}{3}$
 $\frac{4}{4^{\frac{1}{3}}} = 4^{\frac{2}{3}}$
 $\sqrt[3]{4^2}$
 $\frac{\sqrt[3]{16}}{\sqrt[3]{8} \sqrt[3]{2}}$
 $2 \sqrt[3]{2}$

20. $\frac{\sqrt[3]{7^3}}{\sqrt{7}}$
 $\frac{7^{\frac{3}{2}}}{7^{\frac{1}{2}}}$
 $\frac{3}{2} - \frac{1}{2} = \frac{2}{2} = 1$
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21. $\left(\frac{x^{-\frac{2}{3}}}{y^{-\frac{1}{3}}}\right)^{15}$
 $\left(\frac{y^{\frac{1}{3}}}{x^{\frac{2}{3}}}\right)^{15}$
 $\frac{y^{(1/3)15}}{x^{(2/3)15}} = \frac{y^5}{x^{10}}$

22. $\frac{\sqrt{x^3}}{\sqrt[3]{x^2}}$
 $\frac{x^{\frac{3}{2}}}{x^{\frac{2}{3}}}$
 $\frac{3}{2} - \frac{2}{3} = \frac{9}{6} - \frac{4}{6} = \frac{5}{6}$
 $x^{\frac{5}{6}} = \sqrt[6]{x^5}$

23. $\left(\frac{16x^7}{81y^{16}}\right)^{\frac{1}{4}}$
 $\frac{\sqrt[4]{16x^7}}{\sqrt[4]{81y^{16}}}$
 $\frac{2x \sqrt[4]{x^3}}{3y^4}$

24. $(9x^4 y^{\frac{1}{4}})^{\frac{3}{2}}$
 $9^{\frac{3}{2}} x^{\frac{12}{2}} y^{\frac{3}{8}}$
 $9^{\frac{3}{2}} x^6 y^{\frac{3}{8}}$
 $\sqrt[3]{9^3} \sqrt[8]{y^3}$
 $27x^6 \sqrt[8]{y^3}$