

6.5 Solving Radical Equations

Steps to Solve Radical Equations	1	Isolate the radical on one side of the equation
	2	Raise each side of the equation to the power of the index to eliminate the radical sign ** For equation of the form $x^{\frac{m}{n}} = k$, raise each side to the reciprocal power (n/m)
	3	Solve the remaining equation
	4	Check for extraneous solutions

Directions: Solve each equation. Be sure to check for extraneous solutions.

1. $\sqrt{x} - 5 = 12$
 $(\sqrt{x})^2 = (17)^2$
 $x = 289$

$\sqrt{289} - 5 = 12$
 $17 - 5 = 12$
 $12 = 12 \checkmark$

2. $3 - \sqrt[4]{m} = 0$
 $(3)^4 = (\sqrt[4]{m})^4$
 $81 = m$

$3 - \sqrt[4]{81} = 0$
 $3 - 3 = 0 \checkmark$
 $0 = 0 \checkmark$

3. $10 + \sqrt{a+7} = 11$
 $(\sqrt{a+7})^2 = (1)^2$
 $a+7 = 1$
 $a = -6$

$10 + \sqrt{-6+7} = 11$
 $10 + \sqrt{1} = 11$
 $10 + 1 = 11$
 $11 = 11 \checkmark$

4. $\sqrt{6w-5} + 10 = 3$
 $(\sqrt{6w-5})^2 = (-7)^2$
 $6w - 5 = 49$
 $6w = 54$
 $w = 9$

$\sqrt{6(9)-5} + 10 = 3$
 $\sqrt{54-5} + 10 = 3$
 $\sqrt{49} + 10 = 3$
 $7 + 10 = 3$
 $17 = 3 \times$

extraneous
 NO solution

5. $\frac{-36}{-9} = \frac{-9(x-12)^{\frac{1}{2}}}{-9}$
 $[4] = [(x-12)^{\frac{1}{2}}]^{\frac{2}{1}}$
 $4^2 = x-12$
 $16 = x-12$
 $28 = x$

$-36 = -9(28-12)^{\frac{1}{2}}$
 $-36 = -9(16)^{\frac{1}{2}}$
 $-36 = -9(4)$
 $-36 = -36 \checkmark$

6. $(7p-1)^{\frac{1}{3}} + 11 = 7$
 $(7p-1)^{\frac{1}{3}} = -4$
 $7p-1 = (-4)^3$
 $7p-1 = -64$
 $7p = -63$
 $p = -9$

$(7(-9)-1)^{\frac{1}{3}} + 11 = 7$
 $(-63-1)^{\frac{1}{3}} + 11 = 7$
 $(-64)^{\frac{1}{3}} + 11 = 7$
 $-4 + 11 = 7$
 $7 = 7 \checkmark$

$$7. -4\sqrt{43-3x} + 18 = -2$$

$$-4\sqrt{43-3x} = -20$$

$$\sqrt{43-3x} = 5$$

$$43-3x = 25$$

$$-3x = -18$$

$$x = 6$$

$$-4\sqrt{43-3(6)} + 18 = -2$$

$$-4\sqrt{43-18} + 18 = -2$$

$$-4\sqrt{25} + 18 = -2$$

$$-4(5) + 18 = -2$$

$$-20 + 18 = -2$$

$$-2 = -2 \checkmark$$

$$8. (\sqrt{x-8} = \sqrt{43-2x})^2$$

$$x-8 = 43-2x$$

$$3x = 51$$

$$x = 17$$

$$\sqrt{17-8} = \sqrt{43-2(17)}$$

$$\sqrt{9} = \sqrt{43-34}$$

$$\sqrt{9} = \sqrt{9} \checkmark$$

$$9. (\sqrt{6m-38}) = (m-5)^2$$

$$6m-38 = (m-5)(m-5)$$

$$6m-38 = m^2-10m+25$$

$$0 = m^2-16m+63$$

$$0 = (m-9)(m-7)$$

$$m = 9 \quad m = 7$$

$$\sqrt{6(9)-38} = 9-5$$

$$\sqrt{54-38} = 4$$

$$\sqrt{16} = 4$$

$$4 = 4 \checkmark$$

$$\sqrt{6(7)-38} = 7-5$$

$$\sqrt{42-38} = 2$$

$$\sqrt{4} = 2$$

$$2 = 2 \checkmark$$

$$10. (\sqrt{51-5y}) = (y-11)^2$$

$$51-5y = (y-11)(y-11)$$

$$51-5y = y^2-22y+121$$

$$0 = y^2-17y+70$$

$$0 = (y-10)(y-7)$$

$$y = 10 \quad y = 7$$

$$\text{extr.} \quad \text{extr.}$$

NO solutions

$$\sqrt{51-5(10)} = 10-11$$

$$\sqrt{1} = -1$$

$$1 \neq -1$$

$$\sqrt{51-5(7)} = 7-11$$

$$\sqrt{16} = -4$$

$$4 = -4 \times$$

$$11. (\sqrt{x+9}) = (\sqrt{x}-1)^2$$

$$x+9 = (\sqrt{x}-1)(\sqrt{x}-1)$$

$$x+9 = x-\sqrt{x}-\sqrt{x}+1$$

$$x+9 = x-2\sqrt{x}+1$$

$$8 = -2\sqrt{x}$$

$$(-4)^2 = (\sqrt{x})^2$$

$$16 = x$$

extraneous

$$\sqrt{16+9} = \sqrt{16}-1$$

$$\sqrt{25} = 4-1$$

$$5 = 3 \times$$

NO solutions

$$12. \sqrt{5x+4} - \sqrt{x} = 4$$

$$(\sqrt{5x+4} = 4 + \sqrt{x})^2$$

$$5x+4 = (4+\sqrt{x})(4+\sqrt{x})$$

$$5x+4 = 16+4\sqrt{x}+4\sqrt{x}+x$$

$$5x+4 = 16+8\sqrt{x}+x$$

$$\frac{4x-12}{4} = \frac{8\sqrt{x}}{4}$$

$$(x-3)^2 = (2\sqrt{x})^2$$

$$x^2-6x+9 = 4x$$

$$x^2-10x+9 = 0$$

$$(x-9)(x-1) = 0$$

$$x = 9 \quad x = 1 \text{ extraneous}$$

$$\sqrt{4(9)+4} - \sqrt{9} = 4$$

$$7-3 = 4$$

$$4 = 4 \checkmark$$

$$\sqrt{4(1)+4} - \sqrt{1} = 4$$

$$3-1 = 4$$

$$2 \neq 4$$

$$13. \sqrt{3x+1} - \sqrt{x+1} = 2$$

$$(\sqrt{3x+1}) = (2 + \sqrt{x+1})^2$$

$$3x+1 = (2+\sqrt{x+1})(2+\sqrt{x+1})$$

$$3x+1 = 4+2\sqrt{x+1}+2\sqrt{x+1}+x+1$$

$$3x+1 = 5+x+4\sqrt{x+1}$$

$$\frac{2x-4}{2} = \frac{4\sqrt{x+1}}{2}$$

$$(x-2)^2 = (2\sqrt{x+1})^2$$

$$x^2-4x+4 = 4(x+1)$$

$$x^2-4x+4 = 4x+4$$

$$x^2-8x = 0$$

$$x(x-8) = 0$$

$$x = 0 \quad x = 8$$

$$\sqrt{0+1} - \sqrt{0+1} = 2$$

$$1-1 = 2$$

$$0 \neq 2$$

$$x = 8$$

$$\sqrt{24+1} - \sqrt{9} = 2$$

$$\sqrt{25} - 3 = 2$$

$$5-3 = 2$$

$$2 = 2 \checkmark$$