6.6 Function Operations

Operations With Functions	You can add, subtract, multiply, and divide functions.
	Example: If $f(x) = 3x - 7$ and $g(x) = 13 - 2x$, find $f(x) + g(x)$: $3x-7+13-2x = x+6$
	This new function is denoted as: $(f+g)(x)$

All Operations		
	SUM	$(f+g)(x) = f(x) + g(x)$
	DIFFERENCE	$(f-g)(x) = f(x) - g(x)$
	PRODUCT	$(f \cdot g)(x) = f(x) \cdot g(x)$
restricts the domain →	***QUOTIENT**	$\left(\frac{f}{g}\right)(x) = f(x)/g(x)$ $\quad g(x) \neq 0$

Directions: Given $f(x) = x^2 - 8x + 4$, $g(x) = 4x - 3$, and $h(x) = x + 2$, find each function. Indicate any restrictions in the domain.

1. $(f+g)(x)$

$x^2-8x+4+4x-3$

x^2-4x+1

2. $(f-h)(x)$

$(x^2-8x+4)-(x+2)$

$x^2-8x+4-x-2$

x^2-9x+2

3. $(h \cdot g)(x)$

$(x+2)(4x-3)$

$4x^2-3x+8x-6$

$4x^2+5x-6$

4. $\left(\frac{f}{h}\right)(x)$

$\frac{x^2-8x+4}{x+2}$ $\quad x \neq -2$

5. $\left(\frac{h}{g}\right)(x)$

$\frac{(x+2)}{(4x-3)}$ $\quad x \neq 3/4$

6. $(f \cdot g)(x)$

$(x^2-8x+4)(4x-3)$

$4x^3-3x^2-32x^2+24x+16x-12$

$4x^3-35x^2+40x-12$

Directions: Given $f(x) = 2x^2 - x - 12$ and $g(x) = x + 7$, find each function value.

7. $(f+g)(-2)$

$2x^2-x-12+x+7$

$2x^2-5$

$2(-2)^2-5$

$2(4)-5 = \boxed{3}$

8. $(f-g)(8)$

$(2x^2-x-12)-(x+7)$

$2x^2-x-12-x-7$

$2x^2-2x-19$

$2(8)^2-2(8)-19$

$2(64)-16-19 = \boxed{93}$

9. $(f \cdot g)(-1)$

$(2x^2-x-12)(x+7)$

$2x^3+14x^2-x^2-7x-12x-84$

$2x^3+13x^2-19x-84$

$2(-1)^3+13(-1)^2-19(-1)-84$

$-2+13+19-84 = \boxed{-54}$

10. $\left(\frac{f}{g}\right)(5)$

$\frac{2x^2-x-12}{x+7} = \frac{2(5)^2-5-12}{5+7}$

$\frac{50-17}{12} = \frac{33}{12} = \boxed{\frac{11}{4}}$

Composition of Functions

Another method to combine functions is called **composition.**
Given $f(x)$ and $g(x)$, the composite function $(f \circ g)(x)$ is defined as:

$f(g(x))$ plug g into function f

Directions: Given $f(x) = x^3 + 8$, $g(x) = x - 1$ and $h(x) = 5x - 3$, find each function.

11. $(f \circ g)(x)$

$(x-1)^3 + 8$

12. $(h \circ g)(x)$

$5(x-1) - 3$
$5x - 5 - 3$
$5x - 8$

13. $(f \circ h)(x)$

$(5x-3)^3 + 8$

14. $(g \circ f)(x)$

$(x^3+8) - 1$
$x^3 + 7$

Directions: Using the same functions above, find each function value.

15. $(g \circ h)(2)$

$(5x-3) - 1$
$5x - 4$
$5(2) - 4 = \boxed{6}$

16. $(h \circ f)(-2)$

$5(x^3+8) - 3$
$5x^3 + 40 - 3$
$5x^3 + 37$
$5(-2)^3 + 37$
$5(-8) + 37 = -40 + 37 = \boxed{-3}$

17. A store is offering a 15% discount on all items. Also, employees get a 20% employee discount. Write composite functions to model the following:

15% off → 85%
20% off → 80%

a. To model taking the 15% discount and then the 20% discount

$.8(.85x) = 0.68x$

b. To model taking the 20% discount and then the 15% discount

$.85(.8x) = 0.68x$

c. If you were an employee, which discount would you take first? Why?

It doesn't matter since the percentages multiply.

18. Your teacher offers to give the whole class a bonus if everyone passes the next math test. The teacher says that she will give everyone a 10-point bonus and increase everyone's grades by 9% of their score.

a. You earned a 75 on the test. Would you rather have the 10-point bonus first and then the 9% increase, or the 9% increase first and then the 10-point bonus?

A → $75 + 10 = 85$
$85 + .09(85) = 92.65$

B → $75 + .09(75) = 81.75$
$81.75 + 10 = 91.75$

10 pts first

b. Is this the best plan for all students? Explain.

A → $(x+10) + .09(x+10)$
$x + 10 + .09x + .9$
$1.09x + 10.9$

B → $(x + .09x) + 10$
$1.09x + 10$

10 pts first is always better by 0.9 points