6.7 Inverse Relations

Inverse Relations

Notation: The inverse relation R is written as R^{-1}.

To find the inverse of a relation: Switch the x and y values

One-To-One Relationship

A function has a **one-to-one relationship** if both the relation and its inverse are functions

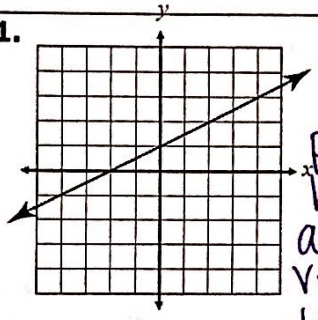
Horizontal Line Test

- **Recall:** The **vertical line test** is used to determine whether a relation is a function given a graph
- The **horizontal line test** is used to determine whether the inverse of a relation is a function
- **BOTH** the relation and its inverse must be a function in order to be a one-to-one function (must pass the horizontal and vertical line test)

Examples

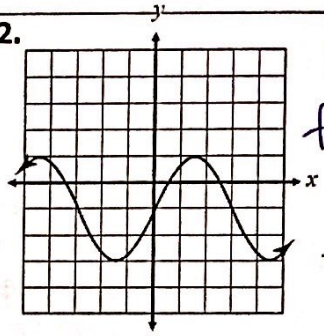
Determine whether the relation shown represents a one-to-one function

1.



yes; passes horizontal and vertical test

2.



no; fails horizontal test

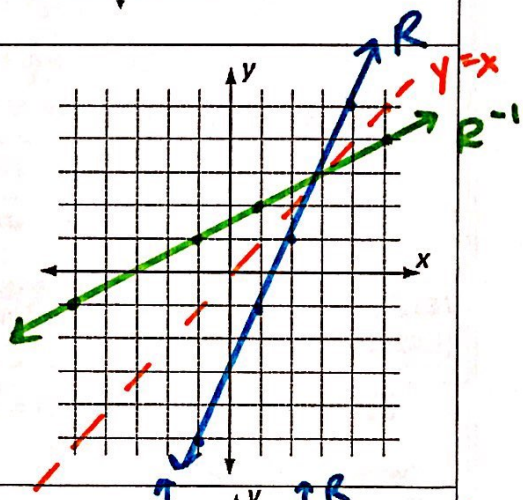
Graphing Inverse Relations

3. Find the inverse realtion, then graph.

Relation R	
x	y
-1	-5
1	-1
2	1
4	5

Inverse R^{-1}	
x	y
-5	-1
-1	1
1	2
5	4

Is R a function? Is the inverse a function? Explain.



4. Graph $y = x^2 + 1$ and its inverse.

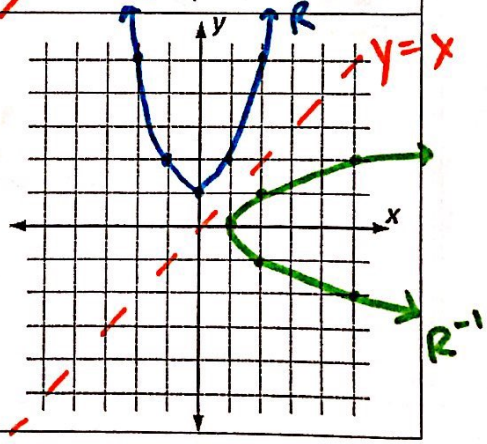
(0,1)

R

X	Y
-2	5
-1	2
0	1
1	2
2	5

inverse R^{-1}

X	Y
5	-2
2	-1
1	0
2	1
5	2



Inverse Functions

Notation: The inverse of a function $f(x)$ is written as $f^{-1}(x)$

To find the inverse of a function:

- **Step 1:** Replace $f(x)$ with y
- **Step 2:** Interchange x with y
- **Step 3:** Solve for y
- **Step 4:** Rewrite y as $f^{-1}(x)$

Examples

Find the inverse of each function

6. $f(x) = 5x$

$y = 5x$

$\frac{x}{5} = \frac{5y}{5}$

$\frac{x}{5} = y$

$f^{-1}(x) = \frac{x}{5}$

7. $f(x) = x - 7$

$y = x - 7$

$x = y - 7$

$+7 \quad +7$

$x + 7 = y$

$f^{-1}(x) = x + 7$

8. $f(x) = \frac{x-8}{3}$

$y = \frac{x-8}{3}$

$x = \left(\frac{y-8}{3}\right)3$

$3x = y - 8$

$3x + 8 = y$

$f^{-1}(x) = 3x + 8$

9. $f(x) = -\frac{2}{5}x + 1$

$y = \frac{-2}{5}x + 1$

$x = \frac{-2}{5}y + 1$

$x - 1 = \frac{-2}{5}y$

$5(x-1) = -2y$

$\frac{5x-5}{-2} = y$

$f^{-1}(x) = \frac{-5}{2}x + \frac{5}{2}$

10. $f(x) = x^2 - 9$

$y = x^2 - 9$

$x = y^2 - 9$

$x + 9 = y^2$

$\pm\sqrt{x+9} = y$

$f^{-1}(x) = \pm\sqrt{x+9}$

11. $f(x) = (x+4)^3 - 6$

$y = (x+4)^3 - 6$

$x = (y+4)^3 - 6$

$x + 6 = (y+4)^3$

$\sqrt[3]{x+6} = y + 4$

$\sqrt[3]{x+6} - 4 = y$

$f^{-1}(x) = \sqrt[3]{x+6} - 4$

Verifying Inverses (Using Compositions)

Two functions f and g, are inverse functions if and only if:

$f \circ g(x) = x$ **AND** $g \circ f(x) = x$

Examples

Determine whether The pair of functions Are inverse functions

12. $f(x) = x^2 + 8$ and $g(x) = \sqrt{x-8}$

$f \circ g(x) = (\sqrt{x-8})^2 + 8 = x - 8 + 8 = x$ ✓

$g \circ f(x) = \sqrt{(x^2+8)-8} = \sqrt{x^2} = x$ ✓

yes, they are inverses

13. $f(x) = -4x + 5$ and $g(x) = \frac{x}{4} - 5$

$f \circ g(x) = -4\left(\frac{x}{4} - 5\right) + 5$

$= -x + 20 + 5$

$= -x + 25$ ✗

no, they are not inverses