

Solving Exponential Equations

1. Use the properties of exponents to **SIMPLIFY** each side of the equation
2. Rewrite the equation so both sides have the **SAME BASE**
3. Drop the bases and **SET THE EXPONENTS EQUAL TO EACH OTHER.**

Type 1- Equations with a Common Base

1. $2^{x+1} = 2^9$

$$x+1=9$$

$$x=8$$

2. $5^{4n+5} = 5^{n-7}$

$$4n+5=n-7$$

$$3n=-12$$

$$n=-4$$

Type 2- Equations without a Common Base

3. $6^{2x-10} = 36$

$$6^{2x-10} = 6^2$$

$$2x-10=2$$

$$2x=12$$

$$x=6$$

4. $2^{p-7} = 8$

$$2^{p-7} = 2^3$$

$$p-7=3$$

$$p=10$$

5. $27^{2x+6} = 3^{2x}$

$$3^{3(2x+6)} = 3^{2x}$$

$$6x+18=2x$$

$$\frac{18}{-4} = \frac{-4x}{-4}$$

$$-\frac{9}{2} = x$$

6. $4^{y+2} = 16^{y-3}$

$$4^{y+2} = 4^{2(y-3)}$$

$$y+2=2y-6$$

$$8=y$$

What if a common base is NOT possible?

1. **ISOLATE** the exponential expression.
2. **TAKE THE LOG** of both sides
3. You may need to **EXPAND** the log (Use the power rule)

cannot take log of a neg.

Type 3- Equations with NO POSSIBLE Common Base

7. $2^x = 61$

$$\log 2^x = \log 61$$

$$\frac{x \cdot \log 2}{\log 2} = \frac{\log 61}{\log 2}$$

$$x = 5.931$$

8. $4^{3x} - 5 = 3$ $4^{3x} = 8$

$$\log 4^{3x} = \log 8$$

$$\frac{3 \times \log 4}{3 \log 4} = \frac{\log 8}{3 \log 4}$$

$$x = 0.5$$

9. $4 \cdot 3^x + 15 = 359$

$$4 \cdot 3^x = 344$$

$$3^x = 86$$

$$\log 3^x = \log 86$$

$$\frac{x \log 3}{\log 3} = \frac{\log 86}{\log 3}$$

$$x = 4.055$$

10. $8 \cdot 11^{7k} - 3 = 213$

$$8 \cdot 11^{7k} = 216$$

$$11^{7k} = 27$$

$$\log 11^{7k} = \log 27$$

$$\frac{7k \log 11}{7 \log 11} = \frac{\log 27}{7 \log 11}$$

$$k = 0.196$$

**Logarithmic Equations
Type 1: LOG=LOG**

1. **CONDENSE** each logarithm.
2. **Use the One-to-One Property:** If $\log_b m = \log_b n$, then
3. **SOLVE** and **CHECK FOR EXTRANEIOUS SOLUTIONS.**

Type 1: LOG=LOG

11. $\log_5(5x+9) = \log_5(6x)$

$$5x+9 = 6x$$

$$\boxed{9 = x}$$

$$\log(5x+9) = \log(6x)$$

$$\log(54) = \log(54)$$

12. $3 \cdot \log_7 4 = \log_7(4x-8)$

$$\log_7 4^3 = \log_7(4x-8)$$

$$64 = 4x - 8$$

$$72 = 4x$$

$$\boxed{18 = x}$$

$$\log_7 64 = \log_7(4(18)-8)$$

13. $\log_4 68 - \log_4 4 = \log_4(3n+11)$

$$\log_4 \frac{68}{4} = \log_4(3n+11)$$

$$17 = 3n+11$$

$$6 = 3n$$

$$\boxed{2 = n}$$

$$\log_4 17 = \log_4(6+11)$$

14. $\log 2 + \log(k^2) = \log(k^2+16)$

$$\log 2k^2 = \log(k^2+16)$$

$$2k^2 = k^2+16$$

$$k^2 = 16$$

$$\boxed{k = \pm 4}$$

$$\log 2(16) = \log(16+16)$$

**Logarithmic Equations
Type 2: LOG=NUMBER**

1. **CONDENSE** and **ISOLATE** the logarithm.
2. Write the equation in **EXPONENTIAL FORM**
3. **SOLVE** and **CHECK FOR EXTRANEIOUS SOLUTIONS.**

Type 1: LOG=NUMBER

15. $\log_2(x-4) = 6$

$$2^6 = x-4$$

$$64 = x-4$$

$$\boxed{68 = x}$$

$$\log_2(64) = 6 \checkmark$$

16. $\log_6(4x+8) - 7 = -3$

$$\log_6(4x+8) = 4$$

$$6^4 = 4x+8$$

$$1296 = 4x+8$$

$$1288 = 4x$$

$$\boxed{322 = x}$$

17. $\log(2x) + \log(x-5) = 2$

$$\log(2x)(x-5) = 2$$

$$10^2 = (2x)(x-5)$$

$$100 = 2x^2 - 10x$$

$$0 = 2x^2 - 10x - 100$$

$$0 = 2(x^2 - 5x - 50)$$

$$(x-10)(x+5)$$

$$\boxed{x=10} \text{ ext}$$

18. $2 \cdot \log x - \log 4 = 2$

$$\log x^2 - \log 4 = 2$$

$$\log \frac{x^2}{4} = 2$$

$$10^2 = \frac{x^2}{4}$$

$$100 = x^2/4$$

$$400 = x^2$$

$$\pm 20 = x$$

$$\boxed{x=20}$$