7.6 Natural Logarithms

What is "e"?

- e is an irrational number with an approximate value of 2.718
- e often occurs as the base of exponential and logarithmic functions that describe real-world scenarios.

Base "e" Exponential Functions

- Exponential functions with base e are called natural base exponential functions
- Example: $f(x) = e^x$

Base "e" Logarithmic Functions

- Logarithmic functions with base e are called natural logs
- Example: $\log_e x$. This is often abbreviated as $\ln x$

Converting Between Forms

Write each equation in logarithmic form.

1. $e^x = 24$ → $\ln 24 = x$
2. $e^9 = x$ → $\ln x = 9$
3. $e^{x+5} = 72$ → $\ln 72 = x + 5$

Write each equation in exponential form.

4. $\ln x = 58$ → $e^{58} = x$
5. $\ln 6 = x$ → $e^x = 6$
6. $\ln(x - 9) = 32$ → $e^{32} = x - 9$

Simplifying With Properties

Condense each expression into a single logarithm.

7. $\ln 3 + \ln 16$ → $\ln(3 \cdot 16)$ → $\ln(48)$
8. $\ln 63 - 2 \cdot \ln 3$ → $\ln 63 - \ln 3^2$ → $\ln \frac{63}{9}$ → $\ln 7$
9. $\frac{1}{3} \cdot \ln 64 + 2 \cdot \ln x$ → $\ln 64^{1/3} + \ln x^2$ → $\ln 4 + \ln x^2$ → $\ln 4x^2$

Expand each logarithm.

10. $\ln 5x$ → $\ln 5 + \ln x$
11. $\ln\left(\frac{a^3}{b}\right)^2$ → $2\ln \frac{a^3}{b}$ → $2(\ln a^3 - \ln b)$ → $2(3\ln a - \ln b)$ → $6\ln a - 2\ln b$
12. $\ln \sqrt[3]{m^2 n}$ → $\ln m^{2/3} n^{1/3}$ → $\ln m^{2/3} + \ln n^{1/3}$ → $\frac{2}{3}\ln m + \frac{1}{3}\ln n$

Solving Equations

Solve each equation below. Check for extraneous solutions.

13. $\ln(4x - 27) = \ln(15 - 2x)$

$4x - 27 = 15 - 2x$

$6x = 42$

$x = 7$ ✓

14. $2 \cdot \ln k = \ln(2k + 15)$

$\ln k^2 = \ln(2k + 15)$

$k^2 = 2k + 15$

$k^2 - 2k - 15 = 0$

$(k - 5)(k + 3) = 0$

$\boxed{k = 5}$ $k = -3$ ext

15. $\ln 72 - \ln 4 = \ln 6 + \ln(a - 2)$

$\ln \frac{72}{4} = \ln 6(a - 2)$

$\ln 18 = \ln(6a - 12)$

$18 = 6a - 12$

$30 = 6a$

$5 = a$ ✓

16. $2 \cdot \ln(m + 4) = \ln 4$

$\ln(m+4)^2 = \ln 4$

$\sqrt{(m+4)^2} = \sqrt{4}$

$m + 4 = \pm 2$

$m = -4 \pm 2$ $-4+2$ $-4-2$

$m = -6, \boxed{-2}$ ext

17. $\ln 8x = 2$

$e^2 = 8x$

$\frac{7.389 = 8x}{8}$

$x = 0.924$

18. $\ln x - \ln 9 = 7$

$\ln \frac{x}{9} = 7$

$e^7 = \frac{x}{9}$

$1096.633 = \frac{x}{9}$

$9869.698 = x$

19. $e^x = 57$

$\ln e^x = \ln 57$

$x \ln e = \ln 57$

$x = 4.043$

20. $e^{x+3} - 6 = 24$

$e^{x+3} = 30$

$\ln e^{x+3} = \ln 30$

$(x+3) \ln e = \ln 30$

$x + 3 = 3.401$

$x = 0.401$

21. $\frac{5e^{4x}}{5} = \frac{95}{5}$

$e^{4x} = 19$

$\ln e^{4x} = \ln 19$

$4x \ln e = \ln 19$

$\frac{4x = 2.944}{4}$

$x = 0.736$

22. $2e^{x-9} + 3 = 87$

$\frac{2e^{x-9}}{2} = \frac{84}{2}$

$e^{x-9} = 42$

$\ln e^{x-9} = \ln 42$

$(x-9) \ln e = \ln 42$

$x - 9 = 3.738$

$x = 12.738$