

# 8.1 Inverse & Direct Variation

Variation is a special relationship between variables.

## Variation Models

### Direct Variation

$$y = kx$$

"y is directly proportional to x"

### Joint Variation

$$y = kxz$$

"y varies jointly with x and z"

$$y = \frac{k}{x}$$

"y is inversely proportional to x"

**K is the CONSTANT OF VARIATION**

## Identifying Equations

**Directions:** Determine whether the equation represents a direct, joint, or inverse variation. Identify the constant of variation.

1.  $y = 2x$

direct;  $k=2$

2.  $y = \frac{24}{x}$

~~direct~~ inverse;  $k=24$

3.  $y = 5xz$

joint;  $k=5$

4.  $3y = x$

$$y = \frac{x}{3}$$

direct;  $k = \frac{1}{3}$

5.  $xy = 36$

$$y = \frac{36}{x}$$

~~direct~~ inverse;  $k=36$

6.  $\frac{4}{3}y = xz$

$$y = \frac{3}{4}xz$$

joint;  $k = \frac{3}{4}$

7.  $\frac{y}{x} = \frac{2}{5}$

$$y = \frac{2}{5}x$$

direct;  $k = \frac{2}{5}$

8.  $2A = bh$

$$A = \frac{1}{2}bh$$

joint;  $k = \frac{1}{2}$

## Direct Variation Examples

9. If y is directly proportional to x and  $y = 28$  when  $x = 7$ , find x when  $y = 52$ .

$$y = kx$$

$$28 = k(7)$$

$$4 = k$$

$$y = 4x$$

$$52 = 4x$$

$$13 = x$$

10. If r is directly proportional to the square of s and  $r = 72$  when  $s = 12$ , find r when  $s = 8$ .

$$r = ks^2$$

$$72 = k(12)^2$$

$$\frac{1}{2} = k$$

$$r = \frac{s^2}{2}$$

$$r = \frac{8^2}{2}$$

$$r = \frac{64}{2} = 32$$

11. The dollar amount  $d$  that Mgean earns varies directly with the number of hours  $h$  that she works. In her last paycheck, she earned \$148.50 working 18 hours. If her next paycheck is \$90.75, how many hours did she work?

$$d = kh$$

$$148.5 = k(18)$$

$$k = 8.25$$

$$90.75 = 8.25h$$

$$h = 11 \text{ hours}$$

### Joint Variation Examples

12. If  $y$  varies jointly with  $x$  and  $z$  and  $y = 48$  when  $x = 8$  and  $z = 15$ , find  $y$  when  $x = 14$  and  $z = 20$ .

$$y = Kxz$$

$$48 = K(8)(15)$$

$$\frac{2}{5} = K$$

$$y = \frac{2xz}{5}$$

$$y = \frac{2(14)(20)}{5}$$

$$y = 112$$

13. If  $p$  varies jointly as  $q$  and the cube of  $r$ , and  $p = 270$  when  $q = 6$  and  $r = 3$ , find  $p$  when  $q = 3$  and  $r = 5$ .

$$p = Kqr^3$$

$$270 = K(6)(3^3)$$

$$\frac{5}{3} = K$$

$$p = \frac{5qr^3}{3}$$

$$p = \frac{5(3)(5^3)}{3}$$

$$p = 125$$

14. For Tom's savings account, the interest earned varies jointly with the principal and amount of time (in years) since opening the account. If Tom opened the account 3 years ago with \$1,200 and earned \$144 in interest, how much interest will he have earned 8 years after opening the account?

$$I = Kpt$$

$$144 = K(1200)(3)$$

$$0.04 = K$$

$$I = 0.04pt$$

$$I = 0.04(1200)(8)$$

$$= \$384$$

### Inverse Variation Examples

15. If  $y$  is inversely proportional to  $x$  and  $y = 7.5$  when  $x = 8$ , find  $x$  when  $y = 5$ .

$$y = \frac{K}{x}$$

$$7.5 = \frac{K}{8}$$

$$60 = K$$

$$y = \frac{60}{x}$$

$$5 = \frac{60}{x}$$

$$x = 12$$

16. If  $m$  is inversely proportional to the square root of  $n$  and  $m = 27$  when  $n = 16$ , find  $m$  when  $n = 64$ .

$$m = \frac{K}{\sqrt{n}}$$

$$27 = \frac{K}{\sqrt{16}}$$

$$108 = K$$

$$m = \frac{108}{\sqrt{n}}$$

$$m = \frac{108}{\sqrt{64}}$$

$$m = 13.5$$

17. The time it takes Adam to drive the Disney World varies inversely with his average rate of speed. If it takes Adam 10.8 hours at an average speed of 50 miles per hour, how long would it take him at an average speed of 60 miles per hour?

$$t = \frac{K}{s}$$

$$10.8 = \frac{K}{50}$$

$$K = 540$$

$$t = \frac{540}{s}$$

$$t = \frac{540}{60}$$

$$t = 9 \text{ hours}$$

### Combined Variation

A **combined variation** is a relationship that contains **both** direct and inverse variation.

### Examples

18. "y varies directly as x and inversely as z"

$$y = \frac{Kx}{z}$$

19. "p varies inversely with q squared and directly with r cubed"

$$p = \frac{Kr^3}{q^2}$$

20. a varies directly as b and inversely as c. If  $a = 16$  when  $b = 12$  and  $c = 6$ , find  $a$  when  $b = 28$  and  $c = 4$ .

$$a = \frac{Kb}{c}$$

$$16 = \frac{K(12)}{6}$$

$$K = 8$$

$$a = \frac{8b}{c}$$

$$a = \frac{8(28)}{4}$$

$$a = 56$$