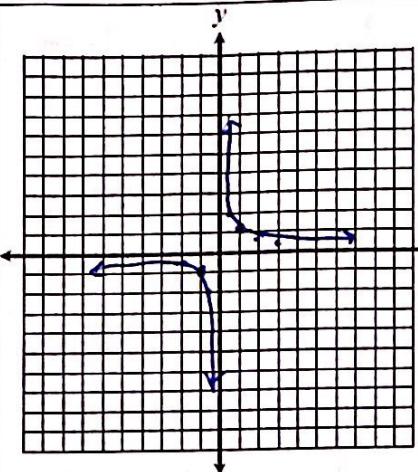


3.2 Reciprocal Functions

RECIPROCAL

Parent Function

$$f(x) = \frac{1}{x}$$



Type of Graph: hyperbola

Domain: $\mathbb{R} \ x \neq 0$

Range: $\mathbb{R} \ y \neq 0$

Vertical Asymptote (VA): $x = 0$

Horizontal Asymptote (HA): $y = 0$

STANDARD FORM

Of a Reciprocal Function

$$f(x) = \frac{a}{x-h} + k$$

$$D: \mathbb{R} \ x \neq h$$

$$R: \mathbb{R} \ y \neq k$$

- h is the **horizontal shift** (+ shifts left, - shifts right)
- k is the **vertical shift** (+ shifts up, - shifts down)
- If a is negative, the function is **reflected** across the **x-axis**.
- $|a| > 1$ represents a vertical **stretch**
- $0 < |a| < 1$ represents a vertical **compression**

VERTICAL ASYMPTOTE:

$$x = h$$

HORIZONTAL ASYMPTOTE:

$$y = k$$

Writing Functions

1. The reciprocal parent function is reflected across the x-axis, then translated 4 units to the right and 3 units down. Write an equation to represent the new function. Identify the asymptotes.

$$VA: x = 4$$

$$f(x) = \frac{-1}{x-4} - 3$$

$$HA: y = -3$$

2. The reciprocal parent function is vertically stretched by a factor of 2, then translated 7 units up and 1 unit left. Write an equation to represent the new function. Identify the asymptotes.

$$VA: x = -1$$

$$f(x) = \frac{2}{x+1} + 7$$

$$HA: y = 7$$

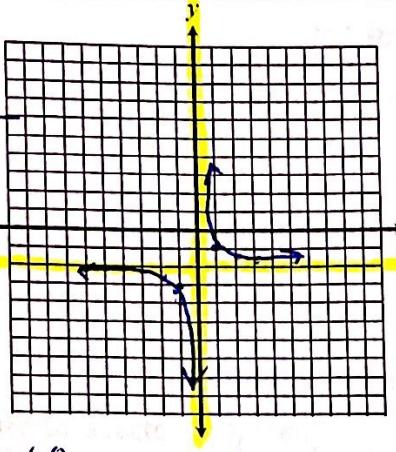
3. A reciprocal function has a vertical asymptote at $x = -2$ and a horizontal asymptote at $y = 5$. Write an equation that could represent this function.

$$f(x) = \frac{1}{x+2} + 5$$

Directions: Graph each function. Identify the domain, range, and asymptotes.

4. $f(x) = \frac{1}{x} - 2$

x	y
-3	-2.3
-2	-2.5
-1	-3
0	-
1	-1
2	-1.5
3	-1.6

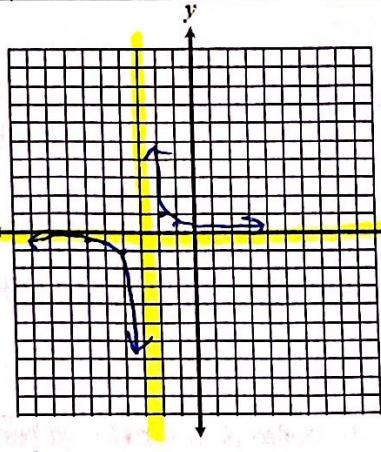


Domain: $\mathbb{R} \setminus x \neq 0$ VA: $x = 0$

Range: $\mathbb{R} \setminus y \neq -2$ HA: $y = -2$

5. $f(x) = \frac{1}{x+3}$

x	y
-4	-.3
-5	-.5
-4	-1
-3	-
-2	1
-1	.5
0	.3

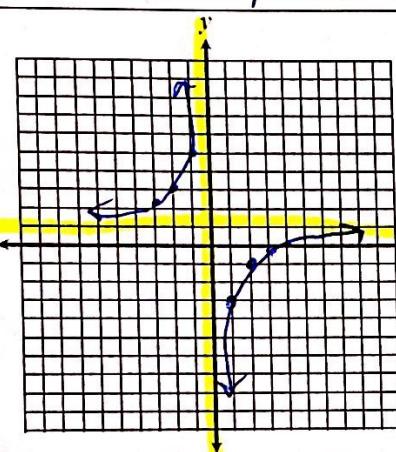


Domain: $\mathbb{R} \setminus x \neq -3$ VA: $x = -3$

Range: $\mathbb{R} \setminus y \neq 0$ HA: $y = 0$

6. $f(x) = \frac{-4}{x} + 1$

x	y
-3	2.3
-2	3
-1	5
0	-
1	-3
2	-1
3	-.3

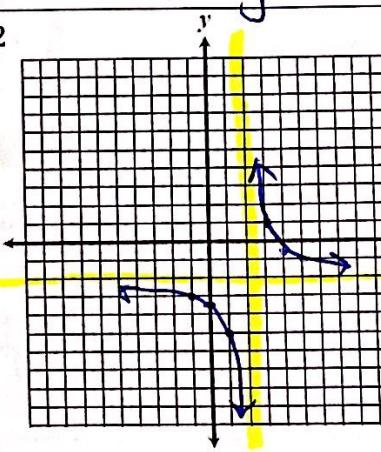


Domain: $\mathbb{R} \setminus x \neq 0$ VA: $x = 0$

Range: $\mathbb{R} \setminus y \neq 1$ HA: $y = 1$

7. $f(x) = \frac{3}{x-2} - 2$

x	y
-1	-3
0	-3.5
1	-5
2	-
3	1
4	-.5
5	-1

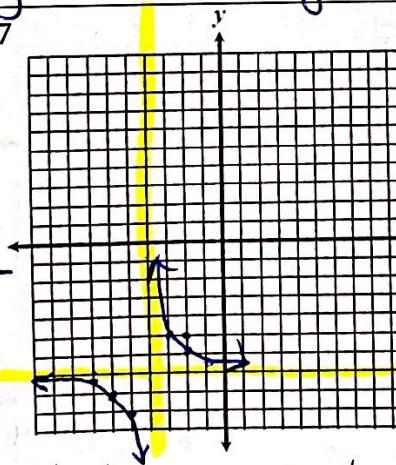


Domain: $\mathbb{R} \setminus x \neq 2$ VA: $x = 2$

Range: $\mathbb{R} \setminus y \neq -2$ HA: $y = -2$

8. $f(x) = \frac{2}{x+4} - 7$

x	y
-7	-7.6
-6	-7
-5	-9
-4	-
-3	-5
-2	-6
-1	-6.3

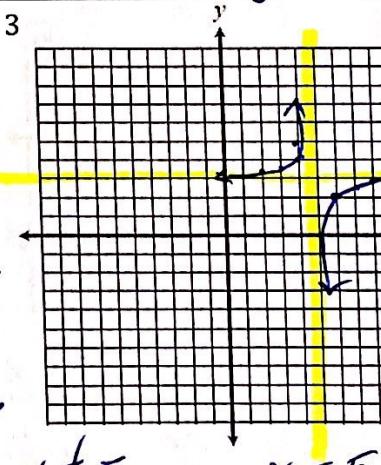


Domain: $\mathbb{R} \setminus x \neq -4$ VA: $x = -4$

Range: $\mathbb{R} \setminus y \neq -7$ HA: $y = -7$

9. $f(x) = \frac{-1}{x-5} + 3$

x	y
2	3.3
3	3.5
4	4
5	-
6	2
7	2.5
8	2.6



Domain: $\mathbb{R} \setminus x \neq 5$ VA: $x = 5$

Range: $\mathbb{R} \setminus y \neq 3$ HA: $y = 3$