

Name: Key

Advanced Algebra II

Date: \_\_\_\_\_

Period: \_\_\_\_\_

Chapter 5: Polynomial Functions

## Chapter 5 Review

Directions: Write each polynomial in standard form. Then, classify by degree and number of terms.

1.  $-9v^2 - 4v - v^4$

$-v^4 - 9v^2 - 4v$

Quartic Trinomial

3. 5

Constant Monomial

2.  $-3 + 9x^3 + x^4 + 2x$

$x^4 + 9x^3 + 2x - 3$

Quartic Polynomial

4.  $2(x^3 + 1) - x^2$

$2x^3 + 2 - x^2$

$2x^3 - x^2 + 2$

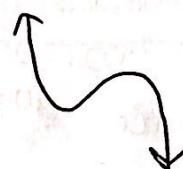
Cubic Trinomial

Directions: Describe the end behavior of each function. Then draw a sketch of what the function could look like using the end behavior and number of turns.

5.  $f(x) = 2x^2 + 12x + 15$

Up-up  
1 turn

6.  $f(x) = -x^3 + 14x^2 - 64x + 97$

Up-down  
2 turns

7.  $f(x) = 3x^7 + 6$

down-up

4 turns



8.  $f(x) = -x^4 + 6x + 2$

down-down

3 turns



Directions: Factor the following completely.

9.  $x^3 - x^2 - 20x = 0$

$x(x^2 - x - 20)$

$x(x - 5)(x + 4)$

10.  $x^3 - 5x^2 - 3x + 15 = 0$

$x^2(x - 5) - 3(x - 5)$

$(x^2 - 3)(x - 5)$

$$11. x^4 - 5x^2 + 4 = 0$$

$$(x^2 - 4)(x^2 - 1)$$

$$(x+2)(x-2)(x+1)(x-1)$$

$$12. x^4 - 5x^2 - 6 = 0$$

$$(x^2 - 6)(x^2 + 1)$$

$$13. x^3 + 64 = 0$$

$$(x+4)(x^2 - 4x + 16)$$

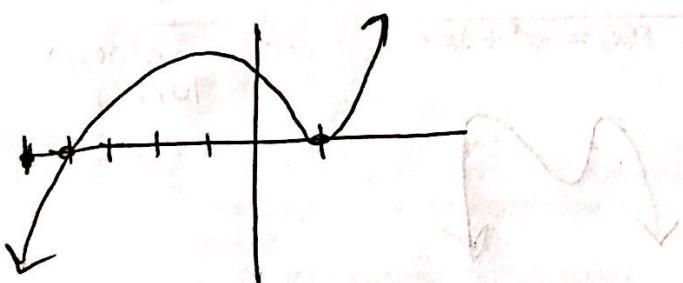
$$14. x^3 - 125 = 0$$

$$(x-5)(x^2 + 5x + 25)$$

Directions: State the zeros and their multiplicities. Then graph the function.

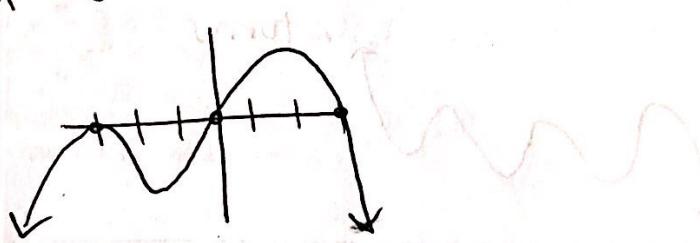
$$15. f(x) = (x-1)^2(x+4)^3$$

$$\begin{aligned} X=1 &\text{ mult 2 bounce} \\ X=-4 &\text{ mult 3 cross} \end{aligned}$$



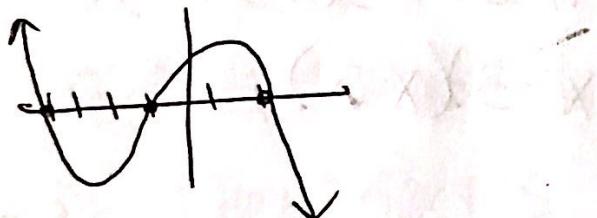
$$16. f(x) = -x(x-3)(x+3)^2$$

$$\begin{aligned} X=0 &\text{ mult 1 cross} \\ X=3 &\text{ mult 1 cross} \\ X=-3 &\text{ mult 2 bounce} \end{aligned}$$



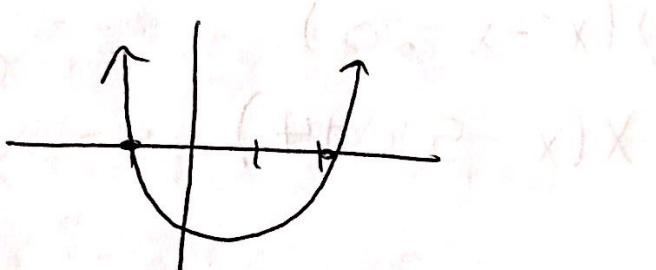
$$17. f(x) = -(x+1)(x-2)(x+4)$$

$$\begin{aligned} X=-1 &\text{ mult 1 cross} \\ X=2 &\text{ mult 1 cross} \\ X=-4 &\text{ mult 1 cross} \end{aligned}$$



$$18. f(x) = (x+1)(x-2)$$

$$\begin{aligned} X=-1 &\text{ mult 1 cross} \\ X=2 &\text{ mult 1 cross} \end{aligned}$$



Directions: Divide using any method.

19.  $(x^3 - 3x^2 - 16x - 72) \div (x - 7)$

$$\begin{array}{r} 7 | 1 & -3 & -16 & -72 \\ + \downarrow & 7 & 28 & 84 \\ \hline 1 & 4 & 12 & \boxed{12} \\ X^2 + 4x + 12 & R \ 12 \end{array}$$

20.  $(4x^3 - 28x^2 - 63x - 82) \div (x - 9)$

$$\begin{array}{r} 9 | 4 & -28 & -63 & -82 \\ + \downarrow & 36 & 72 & 81 \\ \hline 4 & 8 & 9 & \boxed{-1} \\ 4x^2 + 8x + 9 & R \ -1 \end{array}$$

21.  $(x^3 - 5x - 7) \div (x + 1)$

$$\begin{array}{r} -1 | 1 & 0 & -5 & -7 \\ + \downarrow & -1 & 1 & 4 \\ \hline 1 & -1 & -4 & \boxed{-3} \\ X^2 - x - 4 & R \ -3 \end{array}$$

22.  $(4x^3 + x - 2) \div (x + 2)$

$$\begin{array}{r} -2 | 4 & 0 & 1 & -2 \\ + \downarrow & -8 & 16 & -34 \\ \hline 4 & -8 & 17 & \boxed{-36} \\ 4x^2 - 8x + 17 & R \ -36 \end{array}$$

Directions: State the possible rational roots for the following polynomials.

23.  $f(x) = 5x^3 - 17x^2 + 213x - 2$

$$\pm \{1, 2, \sqrt{5}, \frac{2}{\sqrt{5}}\}$$

24.  $f(x) = 3x^7 + 12x^4 + 2x^2 - 12$

$$\pm \{1, 2, 3, 4, 6, 12, \sqrt[3]{3}, \frac{2}{\sqrt[3]{3}}, \frac{4}{\sqrt[3]{3}}\}$$

Directions: Write a function in standard form with the following degrees and zeros.

25. A quartic function with its only real zeros at  $x = -3$  and  $x = 2$

$$\begin{aligned} & x = -3 \quad x = 2 \quad x = i \quad x = -i \\ & ((x+3)(x-2))[(x-i)(x+i)] \\ & (x^2 + x - 6)(x^2 + 1) \\ & x^4 + x^2 + x^3 + x - 6x^2 - 6 \\ & x^4 + x^3 - 5x^2 + x - 6 \end{aligned}$$

26. A cubic function with zeros at  $x = 3i$  and  $x = -1$

$$\begin{aligned} & x = 3i \quad x = -3i \quad x = -1 \\ & ((x-3i)(x+3i))(x+1) \\ & (x^2 - 9(x-9i^2))(x+1) \\ & (x^2 + 9)(x+1) \\ & x^3 + x^2 + 9x + 9 \end{aligned}$$

27. A quartic function with zeros at  $x = 3, x = -4$  and  $x = \sqrt{5}$

$$\begin{aligned} & x = 3 \quad x = -4 \quad x = \sqrt{5} \quad x = -\sqrt{5} \\ & ((x-3)(x+4))[(x-\sqrt{5})(x+\sqrt{5})] \\ & (x^2 + x - 12)(x^2 - \cancel{\sqrt{5}}x + \cancel{\sqrt{5}}x - 5) \\ & (x^2 + x - 12)(x^2 - 5) \\ & x^4 - 5x^2 + x^3 - 5x - 12x^2 + 60 \\ & x^4 + x^3 - 17x^2 - 5x + 60 \end{aligned}$$

28. A quartic function with its only real zeros at  $x = 4$  and  $x = 6$

$$\begin{aligned} & x = 4 \quad x = 6 \quad x = i \quad x = -i \\ & ((x-4)(x-6))(x-i)(x+i) \\ & (x^2 - 10x + 24)(x^2 + 1) \\ & x^4 + x^2 - 10x^3 - 10x + 24x^2 + 24 \\ & x^4 - 10x^3 + 25x^2 - 10x + 24 \end{aligned}$$

Directions: Find all zeros.

29.  $f(x) = 5x^3 - 4x^2 - x$

$$x(5x^2 - 4x - 1)$$

$$x(5x+1)(x-1)$$

$$x=0, -\frac{1}{5}, 1$$

30.  $f(x) = 3x^4 + 5x^3 + 3x^2 + 5x$

$$x(3x^3 + 5x^2 + 3x + 5)$$

$$x[x^2(3x+5) + 1(3x+5)]$$

$$x(x^2+1)(3x+5)$$

$$x=0, \pm i, -\frac{5}{3}$$

31.  $f(x) = 3x^3 + 6x^2 - 19x - 30$

$$\begin{array}{r} 3 \mid 3 & 6 & -19 & -30 \\ \downarrow & -9 & 9 & 30 \\ 3 & -3 & -10 & 10 \\ \hline 3x^2 - 3x - 10 \end{array}$$

$$\frac{3 \pm \sqrt{(-3)^2 - 4(3)(-10)}}{2(3)}$$

$$\frac{3 \pm \sqrt{129}}{6}$$

$$x = -3, \frac{3 \pm \sqrt{129}}{6}$$

32.  $f(x) = 2x^3 - 11x^2 + 20x - 11$

$$\begin{array}{r} 1 \mid 2 & -11 & 20 & -11 \\ \downarrow & 2 & -9 & 11 \\ 2 & -9 & 11 & 10 \\ \hline 2x^2 - 9x + 11 \end{array}$$

$$\frac{9 \pm \sqrt{(-9)^2 - 4(2)(11)}}{2(2)}$$

$$\frac{9 \pm \sqrt{-7}}{4}$$

$$x = 1, \frac{9 \pm \sqrt{-7}}{4}i$$

Directions: Use the table to determine the degree of the function.

x	-3	-2	-1	0	1	2	3
y	129	34	1	-6	1	34	129

33.  $\begin{array}{r} \text{4th degree} \\ \text{Quartic} \end{array}$

x	-3	-2	-1	0	1	2	3
y	129	34	1	-6	1	34	129

$$\begin{matrix} -95 & -33 & -7 & +7 & +33 & +95 \\ \checkmark & \checkmark & \checkmark & \checkmark & \checkmark & \checkmark \\ +62 & +26 & +14 & +26 & +62 & \\ \checkmark & \checkmark & \checkmark & \checkmark & \checkmark \\ -36 & -12 & +2 & +36 & \\ \checkmark & \checkmark & \checkmark & \checkmark \\ +24 & +24 & +24 & & \end{matrix}$$

1  
2  
3  
4

x	-3	-2	-1	0	1	2	3
y	28	19	8	1	4	23	64

34.  $\begin{array}{r} \text{3rd degree} \rightarrow \text{Cubic} \\ 1 \\ 2 \\ 3 \end{array}$

x	-3	-2	-1	0	1	2	3
y	28	19	8	1	4	23	64

$$\begin{matrix} -9 & -11 & -7 & +3 & +19 & +41 \\ \checkmark & \checkmark & \checkmark & \checkmark & \checkmark & \checkmark \\ -2 & +4 & +10 & +16 & +22 & \\ \checkmark & \checkmark & \checkmark & \checkmark & \checkmark \\ +6 & +6 & +6 & +6 & +6 & \end{matrix}$$

Directions: Determine the cubic function that is obtained from the parent function  $y = x^3$  after each sequence of transformations.

35. A vertical stretch by a factor of 2; a vertical translation 5 units down; and a horizontal translation 3 units left.

$$y = 2(x+3)^3 - 5$$

36. A reflection across the x-axis; a vertical translation 6 units up; and a horizontal translation 4 units right

$$y = -(x-4)^3 + 6$$

37. A vertical compression of a factor of  $\frac{1}{2}$ ; a vertical translation 5 units down; a horizontal translation 2 units left; and a reflection across the y-axis

$$y = \frac{1}{2}(-x+2)^3 - 5$$

38. A reflection across the y-axis; a vertical stretch by a factor of 3; and a vertical translation 2 units up

$$y = -3(x)^3 + 2$$

Directions: Find all the real zeros for the following.

39.  $y = 3(x-1)^3 + 2$

$$3(x-1)^3 = -2$$

$$(x-1)^3 = -\frac{2}{3}$$

$$x = 1 + \sqrt[3]{-\frac{2}{3}}$$

$$x-1 = \sqrt[3]{-\frac{2}{3}}$$

40.  $y = 5(-x+1)^3 + 10$

$$5(-x+1)^3 = -10$$

$$(-x+1)^3 = -2$$

$$x = 1 - \sqrt[3]{-2}$$

$$-x+1 = \sqrt[3]{-2}$$

$$-x = -1 + \sqrt[3]{-2}$$