1. $y = 5x^3 - 5x$

$5x(5x^2 - 1)$

$5x(x+1)(x-1)$

$\boxed{x = 0, -1, 1}$

2. $y = 12x^3 - 2x^2 - 2x$

$= 2x(6x^2 - x - 1)$

$= 2x(2x - 1)(3x + 1)$

$\boxed{x = 0, \frac{1}{2}, -\frac{1}{3}}$

3. $y = 5x^3 + 6x^2 + x$

$= x(5x^2 + 6x + 1)$

$= x(5x+1)(x+1)$

$\boxed{x = 0, -\frac{1}{5}, -1}$

4. $y = x^3 - 9x^2 + 27x - 27$

$P/q = \pm\{1, 3, 9, 27\}$

3	1	-9	27	-27
+	↓	3	-18	27
	1	-6	9	0

$x^2 - 6x + 9$

$(x-3)(x-3)$ $\boxed{x = 3, 3, 3}$

5. $y = x^3 - 10x - 12$

$P/q = \pm\{1, 2, 3, 4, 6, 12\}$

-2	1	0	-10	-12
+	↓	-2	4	12
	1	-2	-6	0

$x^2 - 2x - 6$

$\frac{2 \pm \sqrt{(-2)^2 - 4(1)(-6)}}{2(1)}$

$\frac{2 \pm \sqrt{4 + 24}}{2}$

$\frac{2 \pm \sqrt{28}}{2} = \frac{2 \pm 2\sqrt{7}}{2}$

$\boxed{x = -2, 1 \pm \sqrt{7}}$

6. $y = 2x^3 + x - 3$

$\pm\{1, 3, \frac{1}{2}, \frac{3}{2}\}$

1	2	0	1	-3
+	↓	2	2	3
	2	2	3	0

$2x^2 + 2x + 3$

$\frac{-2 \pm \sqrt{2^2 - 4(2)(3)}}{2(2)}$

$\frac{-2 \pm \sqrt{4 - 24}}{4}$

$\frac{-2 \pm \sqrt{-20}}{4}$

$\frac{-2 \pm 2\sqrt{5}i}{4}$

$\boxed{x = 1, -\frac{1}{2} \pm \frac{\sqrt{5}}{2}i}$

7. $y = x^3 + 4x^2 + 7x + 28$

$x^2(x+4) \quad 7(x+4)$

$(x^2+7)(x+4)$

$\boxed{x = \pm\sqrt{7}i, -4}$

8. $y = x^4 - 5x^2 - 36$

$(x^2-9)(x^2+4)$

$(x+3)(x-3)(x^2+4)$

$\boxed{x = \pm 3, \pm 2i}$

9. **no** it would have 4 reals and 2 irrational making too many solutions.

10. $\boxed{y = -(x-2)^3 - 3}$

11. $\boxed{y = -4(x+\frac{1}{2})^3}$

12. $\boxed{y = \frac{1}{3}(-x)^3 + 6}$

13. $\boxed{y = -3(x+2)^2 - 2}$

14. $2(x+1)^3 - 3 = 0$

$(x+1)^3 = 3/2$

$x+1 = \sqrt[3]{3/2}$

$\boxed{x = -1 + \sqrt[3]{3/2}}$

15. $-3(x-2)^3 + 24 = 0$

$(x-2)^3 = 8$

$x-2 = \sqrt[3]{8}$

$x-2 = 2$

$\boxed{x = 4}$

16. $-\frac{1}{2}(x+4)^3 - 1 = 0$

$(x+4)^3 = -2$

$x+4 = \sqrt[3]{-2}$

$\boxed{x = -4 + \sqrt[3]{-2}}$

17. $8(-x-2)^3 + 5 = 0$

$(-x-2)^3 = -5/8$

$-x-2 = \sqrt[3]{-5/8}$

$-x = 2 + \sqrt[3]{-5/8}$

$\boxed{x = -2 - \sqrt[3]{-5/8}}$

18. $x=2, x=8, x=i, x=-i$

$(x-2)(x-8)(x^2+1)$

$(x^2-10x+16)(x^2+1)$

$x^4+x^2-10x^3-10x+16x^2+16$

$y = x^4-10x^3+17x^2-10x+16$

19. $x=3, x=-1, x=\pm i$

$(x-3)(x+1)(x^2+1)$

$(x^2-2x-3)(x^2+1)$

$x^4+x^2-2x^3-2x-3x^2-3$

$y = x^4-2x^3-2x^2-2x-3$

20. $x=-2, x=6, x=\pm i$

$(x+2)(x-6)(x^2+1)$

$(x^2-4x-12)(x^2+1)$

$x^4+x^2-4x^3-4x-12x^2-12$

$y = x^4-4x^3-11x^2-4x-12$

21. $x=-3, x=-5, x=\pm i$

$(x+3)(x+5)(x^2+1)$

$(x^2+8x+15)(x^2+1)$

$x^4+x^2+8x^3+8x+15x^2+15$

$y = x^4+8x^3+16x^2+8x+15$